Birla Institute of Technology & Science, Pilani (Pilani Campus)

CS F320 Foundations of Data Science

Mid-semester Examination

Date: 16/03/2023 | Weightage: 30% | Duration: 90 minutes | Max. marks: 60

1. Explain the different stages in the life-cycle of a data science project. [3]

2. Derive Markov's and Chebyshev's inequalities. Using these results, state and derive the Law of Large Numbers (LLN). [3+3+3=9]

3. Find the singular values of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and find the singular value decomposition of A. [2+4=6]

4. a). Explain the different steps involved in principal component analysis (PCA), mathematically.
b). Explain PCA as a constrained optimization problem.
c). Discuss the number of principal components when the number of samples i.e., m are less than and greater than the number of features, n. [5+3+1=9]

5. Explain the difference between the Frequentist and Bayesian approach using the 'sunrise problem'. Using the Bayesian approach, calculate the probability of sunrise, i.e p_θ for the 3rd day, after observing the sun rise for the last 2 days. [Use uniform prior for $p(\theta)$] [2+3=5]

6. Calculate the bias and variance of the estimator 'sample mean' i.e. $\hat{\mu}_m$ for a set of samples $X = \{x_1, x_2, ..., x_m\}$ drawn from a normal distribution, [4]

$$\mathcal{N}(x_i; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}.$$

7. Discuss the maximum likelihood estimate (MLE) and maximum aposteriori point estimate (MAP) approaches for linear regression. [5]

8. Use the Lagrange multiplier method to find the global maximum and minimum values of the function $f(x, y) = x^2 + 2y^2 - 4y$ subject to the constraint $x^2 + y^2 = 9$. [3]

9. Explain the Naive Bayes classifier and how it is used for the spam filtering application. [3]

10. a). Explain mathematically, a convex function and a convex optimization problem.
b). Under what condition a convex optimization problem gives a unique solution? Explain with the help of a graph. [2+1=3]

11. Explain the concept of a dual problem mathematically, and state the Karush-Kuhn Tucker conditions. [4]

12. Differentiate between gradient descent (GD), momentum accelerated GD, and Nesterov momentum based GD. [3]

13. Define a spline mathematically, and find a linear spline for the data: $x = \{1, 2, 4, 8\}, f(x) = \{3, 7, 21, 73\}$. [3]

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