Birla Institute of Technology and Science, Pilani
Mid Semester Exam, Closed Book, Oct 06, 2016
Theory of Computation (CS F315)

50 Marks **Mins 90**

[**Note**: Answer all the parts of each question together. There are two printed pages.]

Q1. What should be the form of Regular Grammar as per Chomsky Hierarchy of languages? Also, prove that corresponding to every regular grammar (as per Chomsky hierarchy) there exist a DFA. [**3M,** approx 5 mins]

Q2. Answer the following two parts with respect to Chomsky Normal Form (CNF): [**2M+5M,** approx 12 mins]

(a) Can you convert any CFG to CNF? Justify your answer.

(b) Use the following Induction Hypothesis to prove that the parse tree corresponding to every string derivation from a grammar in CNF has odd height.

"Let G = (V, ∑, R, S) be a CFG in Chomsky Normal Form. Then for any variable A ϵ (V-∑), if $A \stackrel{*}{\Rightarrow} w$ in K steps, then |w| = (k+1)/2".

Q3. Answer the following parts. S is the start symbol for each grammar given [**2x6 = 12M,** approx 21 mins]

(a) The language accepted by the following grammar is G = {w ϵ (0U1)* | _____}

$S \rightarrow S_1 \mid S_2$
$S_1 \rightarrow 0S_1 \mid 0E$
$S_2 \rightarrow S_21 \mid E1$
$E \rightarrow 0E1 \mid e$

(b) The language accepted by the following grammar is G = {w ϵ (aUb)* | __________}

S → aA | bB | e
A → aAA | bS
B → bBB | aS

(c) The language accepted by the following grammar is G = {w ϵ (0U1)* | __________}

S → 00S | 11S | S00 | S11 | 01S01 | 01S10 | 10S10 | 10S01 | e

(d) Write a CFG for the language L = {w ϵ {0, 1}* | the length of w is even and first half is all 0's}.

[**Note** that the above grammar can be written with only three production rules. 1M will be deducted for every additional rule.]

(e) Corresponding to the following Regular Expression, List all the strings of length less than 4 (i.e. <4) in lexicographical order: **((0U1)*1(0U01)*)**

(f) The number of states in the minimized DFA that accepts the language (over alphabet {a,b}) defined by the following regular expression are: **(aUb)*(aUb)(aUb)***

Q4. State whether the following are TRUE or FALSE. As stated, justify your answer. Just writing TRUE or FALSE will not fetch any marks. [**18M,** approx 32 mins, Marks are distributed as per complexity of each part]

a) **L = { a^nb^m | n ≤ m ≤ 2n and n ≥ 0} is a Context Free Language**. If TRUE, draw a PDA which accepts L. If FALSE, prove it using pumping theorem.

b) **L = { $a^nb^mc^{n+m}$ | m and n are greater than or equal to one}** is a regular language. If TRUE, draw a DFA which accepts L. If FALSE, prove it using pumping theorem.

c) **A DFA can be viewed as a PDA which never operates on its stack**. If TRUE, show how the transitions of DFA can be converted to that of PDA. If FALSE, give a counter example to justify.

d) **Let regular expression R1 = (00*1)*1 and R2 = 1U0(0 U 10)*11. The language of R1 and R2 is same**. If TRUE, make a most trivial NFA for R1. If FALSE, give another regular expression R3 which accepts the same language as that of R1.

e) **If L1 is a regular language and L1-L2 is also a regular language, then L2 should be a regular language**. If TRUE, show that how DFA can be constructed for L1-L2. If FALSE, give a counter example.

Q5. Let $\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots\ldots\ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, i.e. Σ contains all size 3 columns of 0's and 1's. A string of symbols in Σ gives three rows of 0's and 1's. Consider each row be a binary number and let:

L = {w ϵ ∑* | the bottom row of w is the sum of top two rows}.

For example, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in L$, but $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ do not belong to L.

Show that L is a regular language by making a DFA for it.

[**Hint**: *An input word is read bit-triple by bit-triple, and the most significant bit first. You have to take care whether a possible carry-over from the next bit has occurred or not.*]

[**10M,** approx 20 mins]