Set X

Birla Institute of Technology and Science, Pilani
Comprehensive Exam, Part A [Closed Book], Dec 11, 2017
Theory of Computation (CS F351), Set X

60 Marks **2 Hrs**

[Note: (i) WRITE YOUR ANSWERS AT DESIGNATED PLACE IN THE IN-BUILT ANSWER SHEET PROVIDED.
(ii) ALSO, WRITE YOUR SET (as on top left corner of this sheet) CORRECTLY IN THE IN-BUILT ANSWER SHEET.]

Q1. For each of the following 8 parts choose all the correct options. Each part may have multiple options correct.

i) Deterministic CFL's are not closed under:

(A) Union (B) Complementation (C) Concatenation (D) Intersection

ii)Suppose L $\subseteq \Sigma^*$ is a regular language. If every DFA accepting L has atleast n states, then every NFA accepting L has atleast _____________ states:

(A) n (B) $\log_2 n$ (C) 2^n (D) n^2

iii) Which strings are accepted by the NFA given in Fig-1?

(A) aba (B) abab (C) aaabbb (D) ab

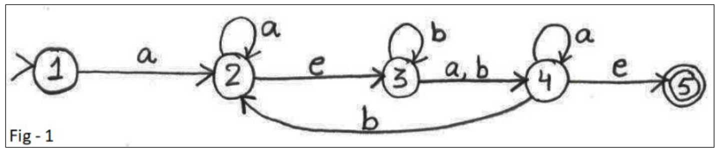

Fig - 1

iv) Which of the following statements (S1, S2, and S3) is/are correct?

S1: Regular expression (01)*0 generates the same language as that of 0(10)*

S2: Regular expression (0U1)*0(0U1)*1(0U1) generates the same language as that of (0U1)*01(0U1)*

S3: Reglar expression (0U1)*01(0U1)*U1*0* generates the same language as that of (0U1)*

(A) S1 (B) S2 (C) S3 (D) All the statements are wrong.

v) Choose the correct statement(s) about the language L = $\{a^nb^n \mid n \geq 0\}$

(A) L is a regular language (B) Minimized DFA for L has two states
(C) L is not a regular language, but it is CFL (D) L is a regular language as well as CFL

vi) Intersection of class-P and class-NP is:

(A) ϕ (B) Class-P (C) Class-NP (D) Class-NP complete

vii) Decidable languages are closed under:

(A) Union (B) Intersection (C) Complementation (D) None of the options given here

viii) Consider the language L = $\{a^nb^n, n \geq 0\}$. As shown below, M_1 is the deterministic PDA accepting L\$ using one look ahead. There are few transitions missing in M_1. Identify them.

M_1=({p, q, A, B, $q_\$$} , {a, b} , {a, b, S} , Δ_1 , p ,{$q_\$$}), Where Δ_1 contains:

((p, e, e) , (q, S)) **((q, a, e) , (A, e))** **((A, e, S), (A, aSb))** **((q, b, e), (B, e))**
((B, e, S), (B, e)) **((q, \$, e), ($q_\$$, e))**

(A) ((A, e, a), (q, e)) (B) ((B, e, b), (q, e)) (C) ((A, e, b), (q, e)) (D) ((B, e, a), (q, e))

Q2. Answer the following parts. Choose the most appropriate option wherever required.

a) A deterministic Finite Automaton is a quintuple (K, Σ, δ, s, F), where δ is a function from __________ to _____.

b) An NFA is a quintuple (K, Σ, δ, s, F), where δ, the transition relation, is a subset of K x ______ x ______.

c) Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as w = xyz, such that $|xy| \leq n$, ___________, and $xy^iz \in L$ for each __________.

d) There is ____ (exponential/polynomial) algorithm which, given an NDFA, constructs an equivalent DFA.

e) There is ____ (exponential/polynomial) algorithm which, given a regular expression, constructs an equivalent NDFA.

f) There is ____ (exponential/polynomial) algorithm which, given two DFA's, decides whether they are equivalent.

g) There is ____ (exponential/polynomial) algorithm which, given two NFA's, decides whether they are equivalent.

h) There is ____ (exponential/polynomial) algorithm which, given a CFG G and a string x, decides whether $x \in L(G)$.

i) The intersection of a CFL with a regular language is a ________ .

j) The class of CFL's is closed under ____________ (concatenation / union / both concatenation and union).

Q3. With respect to three definitions of **clique**, **triangle**, and **language L**, answer the parts that follow:

i. A **clique** in an undirected graph G = (V, E) is a subset of vertices, each pair of which is connected by an edge in G. In other words, a clique is a complete subgraph of G.

ii. A **triangle** in an undirected graph is a 3-clique, i.e. clique with exactly 3 vertices.

iii. **Language L** = {"G" | G is an adjacency matrix representation of an undirected graph G with n vertices that contains a triangle}.

a) Clique problem is undecidable: YES/NO

b) Clique problem is NP-complete: YES/NO

c) Clique problem belongs to class NP: YES/NO

d) Language L is NP-complete: YES/NO

e) __________ (3-SAT / Vertex cover / both 3-SAT and Vertex-cover) is/are polynomial time reducible to Clique.

f) A person gave the following proof to prove that L belongs to class P:

Proof: Let G = (V, E) be an undirected graph with a set V of vertices and a set E of edges. Number the vertices from 1 to n. Enumerate all the triples (t1, t2, t3) with t1, t2, t3 $\in$ V and t1 < t2 < t3 and then check whether or not all three edges (t1, t2), (t1, t3), and (t2, t3) exist in E.

Is the above proof correct?

If YES, complete the following: Enumeration of all triples require O(_________) time; Checking whether or not all three edges belong to E takes O(__________) time. Thus overall time is O(___________).

If NO, briefly justify what is wrong with the proof?

Q4. Complete the production rules of the following CFG's. S is the start symbol and ∑ = {a, b, c} for each.

a)	**b)**
$L_1 = \{b^n a^m c^{m+3} b^{2n+1} \mid n >= 1, m >= 0\}$ S→ bTbb T→ ______| _______ A → aAc | e	**$L2 = \{ a^i b^j c^k \mid i, j, k \geq 0$, and $(i = j$ or $i = k) \}$** S → XY | W X → _________ | ________ Y → cY | e W → aWc | Z Z → _________ | ________

Q5. Answer the following parts:

a) Which of the following two grammars (i.e. G1 and G2) is/are LL(1)? S is the start symbol.

G1: S -> cA | b A -> cBC | bSA | a B -> cc | Cb C -> aS | ba

G2: S -> AaS | B A -> cS | e B -> b

b) Which of the following two grammars (i.e. G3 and G4) is/are ambiguous? S is the start symbol.

G3: S -> +SS | -SS | x

G4: S -> S[S]S | e

Q6. Answer the following parts:

a) Suppose that L is a CFL and R is a Regular Language. Is L - R necessarily context free (YES/NO)? Justify briefly.

b) Suppose that L is a CFL and R is a Regular Language. Is R - L necessarily context free (YES/NO)? Justify briefly.

c) Grammar G1 = (**{S, A, B, a, b}, {a, b}, { S → A | B | a , A → S | B | bA , B → S | A | aB}, S**). Rewrite the rules of G1 after removing all the unit production rules.

d) Grammar G2 = (**{S, B, C, D, a, b}, {a, b}, { S → aCbb , B → CD , C → D | a | e , D → B | b | e}, S**). Rewrite the rules G2 after removing all the null production rules.

Q7. Suppose L1 and L2 are two regular languages. Consider the definition of languages L3 and L4 as follows:

L3 = {w | w ∈ L1 and not all strings of L2 are substrings of w}

L4 = {w | w ∈ L1 and every string of L2 is a substring of w}

a) To prove L4 is regular, can you write an expression to define L4 using each of intersection operation (∩), ∑, Kleene star (*), L1 and L2? If yes, write it. If no, briefly justify that L4 is not regular?

b) To prove L3 is regular, can you write an expression to define L3 using L1, L4, and only one operation out of set-union (U), set-difference (-), set-intersection (∩), or set-complement (¯)? If Yes, write it. If no, briefly justify that L3 is not regular?

c) Suppose language L3 is a result of some operation (say OPER) over regular languages. Can we conclude that regular languages are closed under OPER operation? Why?

Q8. If L is a Regular language, then rev (L) = {w | reverse(w) ϵ L}. With respect to it, answer the following:

a) Complete the following definition for defining the reverse of a Regular Expression R:

- If R is ________Q8 a (i)___________, then rev (R) = R.
- If R is:
  - F U G, then rev (R) = ___________Q8 a (ii)_______________
  - FG, then rev (R) = ___________Q8 a (iii)_______________
  - F*, then rev(R) = ___________Q8 a (iv)_______________

b) Write the reverse of following regular expression R *using the definition in part (a)* above:

R = a(bacb)* U b*(e U c U da*)

Q9. Answer the following parts carefully as there will be no partial marking in these.

a) Make an NFA with exactly three states for the language defined by a regular expression 0*1*0*0.

b) Complete the following proof (and no explanation) to show that class P is closed under union operation.

Suppose that language L1 ∈ P and language L2 ∈ P. Thus, there are polynomial-time TMs M1 and M2 that decide L1 and L2, respectively. Specifically, suppose that M1 has running time $O(n^{k1})$, and that M2 has running time $O(n^{k2})$, where n is the length of the input w, and k1 and k2 are constants. A Turing machine M3 that decides L1 U L2 is as:

M3 = On input w:

1. Run M1 with input w. …………………..

In addition, what is the time complexity of M3 using big-oh notation?

c) Over ∑ = {a, b, c}, let

L1 = {w ∈ (a U b U c)* | w has equal number of a's, b's, and c's. Also, number of a's, b's, and c's > 0}.

L2 = {w | w∈ (abc)(abc)*}

A 2-tape Turing Machine (shown in Fig-2) takes a string w1 ∈ L1 on tape1 and transforms it to another string w2 ∈ L2 onto tape 2. Assume that string w1 always belongs to L1, i.e. the TM semi-decides the language. The TM shown has symbols missing on some of the arrows (and nothing else). Complete the TM. The initial and final configuration of both the tapes is: ►⊔w.

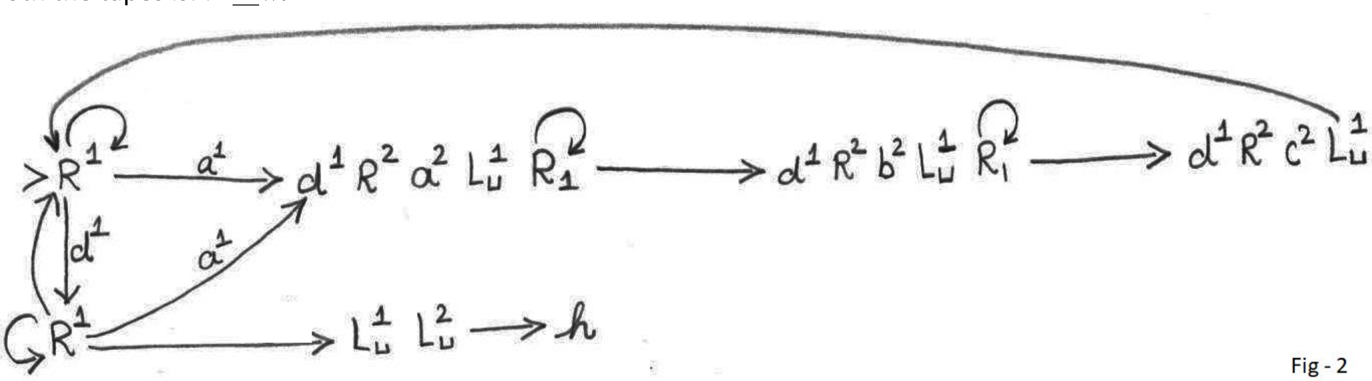

Fig - 2