Birla Institute of Technology and Science, Pilani
Mid Semester Exam, Closed Book, Oct 13, 2017
Theory of Computation (CS F351)

60 Marks **Mins 90**

[**Note**: **Answer all the parts of each question together.**]

Q1. Complete the following proofs:

a) To prove: Regular languages are closed under intersection operation.
Proof: Let L1 and L2 be two regular languages with corresponding DFA's M1 = (Q_1, ∑, δ_1, s_1, F_1) and M2 = (Q_2, ∑, δ_2, s_2, F_2). If DFA M3 = (Q, ∑, δ, s, F) accept language L1 ∩ L2, its tuples (in terms of that of M1 and M2) are.
Q = __________ , δ = ____________ , s = ____________ , F = ______________.

b) Let G1 = (V, ∑, R, S) be a regular grammar with rules as per Chomsky hierarchy of languages. Let $\sum_1 \subset \sum$ be some smaller alphabet. Over $\sum_1$ consider another grammar G2 = (V_1, $\sum_1$, R_1, S_1) such that the language of G2 is a subset of the language of G1, i.e. L(G2) = L(G1) ∩ $\sum_1$*. Prove that G2 is also a Regular grammar.
[**Hint:** For every Regular grammar there exist a DFA.] **[5M + 5M = 10 Marks]**

Q2. State whether the following are TRUE or FALSE with justification. Just writing TRUE/FALSE will not fetch any marks for you.

a) Let L1 be a regular language contained in set **(a U b U c)*** , i.e., regular language L1 ⊆ **(a U b U c)*.** Also, let language L2 contain set of strings which are obtained by rearranging each string of L1 such that all a's appear first, then all b's, and then all c's. ***Language L2 will always be a CFL.***

b) Let L1 be a regular language and N1 is an NFA which accepts L1. Let N2 be another NFA which is same as N1 except that final and non-final states are interchanged. ***The language of N2 will always be complement of L1.***

c) ***If L1 is regular, then L1 is necessarily a CFL.*** To prove (or dis-prove) it use different types of automata done in the course.

d) ***The necessary and sufficient condition to prove that any two languages L1 and L2 are equal is L1' ∩ L2 = ∅.*** Here L1' means complement of L1.

e) ***If L is a non-regular language and L1 is a finite set of strings, then L U L1 will always be non-regular.*** Assume here that the strings in L1 are not in L and ∑ is same for both L and L1.

[2M x 5 = 10M]

Q3. Answer the following parts sequentially with respect to CFG **G = ({S, T, 0, 1}, {0, 1}, R, S)**, where R is given as:

S → SS | 1TT | T1T | TT1 | e

T → 0S | S0S | S0 | 0

a) What is the language accepted by G? *Do it very carefully as the next part is dependent on it.*

b) Make a PDA (graphical representation) for language accepted by G. Here, do not directly convert the CFG to PDA using a theorem given in your text book. Rather, on the basis of answer of Q3(a) above, make a PDA.

c) Reduce the grammar G to Chomsky Normal Form such that the language is L(G) – {e}. Recall that CFG is in CNF if RHS of every production rule is either one terminal or two non-terminals.

d) Is G ambiguous? Is yes, give a smallest length string (and parse trees) which makes it ambiguous. If No, remove the null production rules from the grammar.

[2M + 6M + 3M + 4M = 15M]

Q4. Let L = {w#x | w is a substring of x, where w, x ∈ (a U b)*}. Is L a CFL? If YES, make a PDA for it. If NO, prove it using pumping theorem.

[8M]

Q5. Construct a DFA for the following languages over ∑ = {a, b}:

a) L = {w ∈ ∑* | number of a's in w ≥ 2 and number of b's in w ≤ 1}

b) L = {w ∈ (a U b)* | difference between number of a's and b's in w is not divisible by 5}

[4M + 6M = 10M]

Q6. For Σ = { 0,1,2,3}, construct a CFG for the language containing the set of strings that contain exactly two 1's.

[7M]