8

 $\mathbf{2}$ 

 $\mathbf{5}$ 

 $\mathbf{4}$ 

Note: No marks will be given if the justification for your answer is not provided.

- 1. (a) Let  $L_1$  be the regular language of the regular expression  $a^*b^* + b^*a^*$ . Give an example of a string  $\{a, b\}$  which is not in  $L_1$ .
  - (b) Design a DFA which accepts  $L_1$  (You may consider any method).
- 2. Consider the following NFA (over the alphabet  $\{a, b\}$ )



- (a) What is the shortest string *not* accepted by this NFA?
- (b) Let  $L_1$  denote the set of all strings not accepted by this NFA. Write a regular expression for  $L_1$ .3
- (c) Convert the regular expression of Part (b) to an equivalent NFA.
- 3. Let  $L_1$  and  $L_2$  be regular languages over the alphabet  $\Sigma$ . Define the language  $L_3 = \{\alpha\beta\gamma \mid \alpha\gamma \in L_1, \beta \in L_2\}$ , that is,  $L_3$  is obtained by inserting strings in  $L_2$  inside strings in  $L_1$ . Prove that  $L_3$  is regular too.
- 4. Using Pumping Lemma prove that  $L = \{w : w = w^R, w \in \{0, 1\}^*\}$  is not regular (This is the language of binary palindromes). Where  $w^R$  denotes the reverse of the string w. **10**
- 5. Let  $\alpha$  be a string (over some alphabet  $\Sigma$ ). By  $\operatorname{odd}(\alpha)$ , we refer to the string obtained by deleting symbols at all even positions of  $\alpha$ . That is, if  $\alpha = a_1 a_2 a_3 \ldots a_n$ , then  $\operatorname{odd}(\alpha) = a_1 a_3 a_5 \ldots a'_n$ , where "n'" is n or n-1 according as whether n is odd or even. For a language  $L \subseteq \Sigma^*$ , define  $\operatorname{odd}(L) = \{ odd(\alpha) \mid \alpha \in L \}$ . Prove that if L is regular, then  $\operatorname{odd}(L)$  is regular too. 10
- 6. (a) Give an example of a non-regular language L for which the asterate  $L^*$  is regular.
  - (b) Suppose that  $L_1$  and  $L_2$  are two languages (over the same alphabet) given to you such that both  $L_1$  and  $L_1L_2$  are regular. Prove or disprove:  $L_2$  must be regular too. 6