**BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI**

**CS F351 (Theory of Computation) Mid Semester Exam, 2022 – 23 [Closed Book]**

**November 05, 2022** **90 Minutes** **MM: 60**

**<u>Note</u>: There are seven questions. Answer all parts of a question together. ε denotes empty string.**

**Q1 [10M]**. Consider $F$ a finite language over $\Sigma = \{0, 1\}$. Also, consider $|F| = p$ and $M$ be a Deterministic Finite Automaton with $s$ states such that $L(M) = F$. Prove or disprove $s \geq \log_2(p + 1)$.

**Q2 [10M]**. Let $\Sigma = \{0, 1\}$and language $L = \{\gamma \in \Sigma^* \mid \gamma$ ends with 11 however does not starts with 11$\}$.

- **a)** Construct a DFA which recognizes the language L with maximum seven states. Do not forget to mark the initial state, final states, and dead state (if any).
- **b)** Construct an NFA whose language is L with exactly one **ε** transition.

**Q3 [5M]**. Consider the language $L$ over $\Sigma = \{0,1\}$ and the following language partial (L).

partial (L) = $\{y \mid y \in \Sigma^*$, and there exists $z \in \Sigma^*$ such that $|y| = |z|$ and $yz \in L\}$

*If L is CFL, then partial (L) must also be CFL*. Disprove this statement using the following two facts:

a) Intersection of CFL and RL is a CFL.
b) R = **00*11*00*1** is a regular expression.

**Q4 [6+4 = 10M]**. For the language $P_1$ over $\Sigma = \{0, 1, 2\}$

$$P_1 = \{0^x (12)^y \; x, y \geqslant 0 \text{ and } x \geqslant y\}$$

a) Design a two-state PDA M for recognizing the language $P_1$ by empty stack. Assume that Z0 is already on the stack. Note that zero credits will be given to those students who will utilize more than 2 states.
b) Consider the following incomplete CFG for $P_1$. Complete it by filling the four blanks. S is the start symbol, U, V are the variables. Don't use any extra variable.
S -> U V
U -> _____ | ______
V -> _____ | ______

**Q5 [10M]**. Consider the following CFG for the set of all non-null strings with equal number of a's and b's.

$S \rightarrow aB|bA|SS|aC|bD$, $A \rightarrow a$, $B \rightarrow b$, $C \rightarrow SB$, $D \rightarrow SA$.

Execute the CYK algorithm (as discussed in the class) and identify whether the string **a a b b a b** belongs to the above grammar or not.

**Q6[9M]**. Consider the following CFG for generating the arithmetic expressions involving addition and multiplication operations. Here, A is the start symbol.

$$A \rightarrow a \mid A + A \mid A \times A$$

**(a)** Design *all* the parse trees for the expression a + a × a + a
**(b)** Design an unambiguous grammar to generate the same language. Do not introduce any new variable (other than A). Assume all the operations to be evaluated from left to right.

**Q7 [6M].** Over $\sum = \{a, b\}$, let $L1 = \{a^n b^n \mid n \geq 0\}$. Using pumping lemma for CFL's, the following proof claims to prove that L1 is not a CFL. Is the proof right? Justify your answer.

**<u>Proof</u>**: Suppose L1 is a CFL and p be the pumping length. Let $w = a^p b^p$. From pumping theorem we know that w can be divided in five parts uvxyz such that $uv^ixy^iz \in L1$ for all $i \geq 0$.
Let $v = a^k$ such that $0 < k < p$. Now, $u = a^{p-k}$, $v = a^k$, $x = e$, $y = e$, and $z = b^p$.
Therefore, now for i = 0 the resulting string $w = a^{p-k} b^p \notin L1$.
Therefore, we can conclude that L1 is not a CFL.