BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
CS F351 (Theory of Computation) Comprehensive Exam, 2022 – 23
PART-B [Open Book]

December 30, 2022 **MM: 21**

Q1 [5x3M = 15M]. Are the following languages decidable/un-decidable? If it is decidable, prove it. If not, prove by reduction using the language A_{TM} = {"M","w" | "M" is a Turing Machine and "M" accepts "w"}.

- **a)** L2 = {"M" "w" | M is Turing Machine, w is a string, and some Turing Machine M1 exists such that w does not belong to L(M) ∩ L(M1)}
- **b)** L3 = {"M" | M is a Turing Machine that halts on all inputs and for some undecidable language B, L(M) = L(B)}
- **c)** L4 = {"M" | M is the Turing Machine and M is the only Turing Machine that accepts L(M)}
- **d)** L5 = {"M" | M is a Turing Machine and there exist a TM M1 such that the encodings of M and M1 are not same but L(M) = L(M1)}
- **e)** L6 = {"M" | M is a Turing Machine, and there exists two Turing Machines M1 and M2 such that L(M) ⊆ L(M1) U L(M2)}

Q2 [6M]. Decide whether each of the following statements are True/False. If TRUE, prove formally. If FALSE, give a counter example. For giving the counter example, use the following languages only.

$L_1 = \{a^P \mid P \text{ is prime}\}$
$L_2 = \{a^p : p \text{ is greater than 0 and is not prime}\}$
$L3 = \{a^n b^n \mid n \geq 0\}$

In addition to above, you can consider any number of finite languages.

- **a)** The union of an infinite number of regular languages must be regular.
- **b)** If L_1 and L_2 are not regular languages, then $L_1 \cup L_2$ is not regular.
- **c)** If L^* is regular, then L is regular.