BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
CS F351 (Theory of Computation) Comprehensive Exam, 2023 – 24
PART-B [Open Book]

December 15, 2023 **MM: 23**

Q1 [8M]. Let $w = w_1w_2 \ldots w_{n-1}w_n$ be any string of length n in {0,1}*. We define a language L_w as follows.

$L_w = \{x \mid x = w_i w_{i+1} \ldots w_{j-1} w_j$, and $1 \le i \le j \le n\}$.

Find the minimum number of states in a non-deterministic finite automaton (NFA) that accepts L_w. Prove your result.

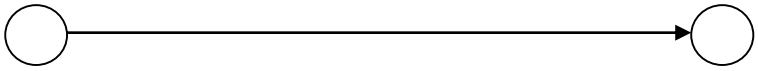
Q2 [8M]. Let N be a nondeterministic Turing machine (NTM). We say that N faces a dilemma if at some point in its working, it encounters a situation where the finite control is in the state p, the head scans the tape symbol a, and δ(p,a) offers multiple (two or more) possibilities, where p is neither the accept nor the reject state. Consider the following language:

$DILEMMA_{ALL}$ = {N | N is an NTM which faces a dilemma at least once on each input }

Prove that $DILEMMA_{ALL}$ is not recursively enumerable.

Q3 [7M]. A two stack PDA is a simple extension of one stack PDA and is defined by a six tuple M = (Q, Σ, Γ, δ, q0, F) where Q is the finite set of states, Σ is the input alphabet, Γ is the stack alphabet (we assume that both stacks use the same alphabet), q0 is the start state, F is the set of final states, and δ is a function from Q x Σ x Γ x Γ to Q x ΓxΓ.

The transitions are labeled as [input symbol or e, top of S1/α, top of S2/β] where S1 and S2 refer to top element of stack 1 and top element of stack 2, respectively, and α and β are the symbols pushed on stack 1 and stack 2, respectively. If one of the stack parts is missing, then the transition is independent of that stack's top symbol and that stack is unchanged. Graphically, following is the representation of **[input symbol or e, top of S1/α, top of S2/β]**



For example, a transition of the form: [0, 1/01, 2/23] means that input symbol is 0, the symbol popped out from first and second stack is 1 and 2 respectively, the symbols pushed in stack 1 and stack 2 are 01 and 23 respectively.

Complete the following deterministic two stack PDA L. Assume that Z0 is the bottom marker symbol for both the stacks and is present in the stack, # is the end marker symbol for the input string.

$L = \{a^n b^{2n} c^{4n} \mid n >= 1\}$