Design and Analysis of Algorithms (CS F364) Comprehensive Examination Part B (Second Semester, 2022)

There are 3 questions in all and total marks are $15 + 15 + 20 = 50$. Please show all steps in computations or proofs. This is an **open book exam**. You can use any printed or handwritten material. Calculators are allowed. Time: 105 minutes.

1. We define a variant of the 3SAT language as follows:

   $\texttt{LOCAL-3SAT} = \{\, \Phi(x_1, \ldots, x_n) \mid \Phi$ is a satisfiable Boolean formula in 3CNF form such that each clause involves variables numbered ± 10 of each other $\}$.

   Prove or disprove

   $$\texttt{LOCAL-3SAT} \in \texttt{P}.$$

2. *Linear insertion* sort can sort an array of numbers in place. The first and second numbers are compared; if they are out of order, they are swapped so that they are in sorted order. The third number is then placed in the appropriate place in the sorted order. It is first compared with the second; if it is not in the proper order, it is swapped and compared with the first. Iteratively, the k'th number is handled by swapping it downward until the first k numbers are in sorted order. Determine the expected number of swaps that need to be made with a linear insertion sort when the input is a random permutation of n distinct numbers.

3. A *semiprime* number is a number of the form $n = pq$, where p and q are primes. Design a polynomial-time algorithm that takes as input a semiprime n and $\phi(n)$, and gives as output its prime factors p and q. Find the time complexity of your algorithm. Show the working of your algorithm for $n = 10403$ and $\phi(n) = 10200$.