BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI (RAJASTHAN)

Second Semester, 2021-2022

60 Marks (30% Weight) **Mid-Semester Test** **Open Book**

Course Number: CS F364 **Course Title:** Design & Analysis of Algorithms
Date **:** March 09, 2022 **Time** **:** 09.00 AM - 10.30 AM

Note: There are four questions in all. Please answer all parts of a question in sequence and in continuation.

Q1. Solve the following:

1.1 Find Omega *(Ω)* notation of function $f(n)=2n^2 + 6\,n * log_2(n) + 6n$.

1.2 Find upper bound of the recurrence $T(n) = T(n/2) + n$.

1.3 Find upper bound of the recurrence $T(n) = T(n/2) + 1$.

1.4 Using greedy method, trace the graph *[(a, b, 3), (a, d, 7), (b, c, 4), (b, d, 2), (c, d, 5), (c, e, 6), (d, e, 4)]*, (where the ordered-triplet *(x, y, N)* means the cost or weight along the directed edge *(x, y)* is *N*) to get shortest path from vertex 'a' to all other vertices.

1.5 Prove that $100n + 5$ is $O(n^2)$ and specify the values of the constant *C* and the n_0.

Character	Frequency
C	260
E	240
T	116
B	60
V	44
S	20
U	18
N	16
R	10

Table Q2.1

Marks Q1 [3 x 5 = 15]

Q2. Solve the following:

2.1 Find Huffman codes for the frequency table given in the Table Q2.1.

2.2 Using Dynamic Programming, solve the Knapsack problem, given: *n=3*, $\{W_1, W_2, W_3\}=\{1, 2, 2\}$, $\{P_1, P_2, P_3\}=\{18, 16, 6\}$ *and Capacity=4*.

2.3 Solve the matrix multiplication as given in the Table Q2.3 using Strassen's algorithm.

$$\begin{bmatrix} 1 & 3 & 2 & 5 \\ 1 & 0 & 1 & 4 \\ 4 & 1 & 0 & 1 \\ 5 & 2 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 2 \\ 3 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Table Q2.3

Marks Q2 [5 x 3 = 15]

Q3. Given the transport network as *[(S,a, 4,3), (S,b, 7,2), (a,c, 5,2), (a,D, 3,1), (b, d, 2, 1), (b, D, 1, 1), (c, D, 2, 2), (d, D, 1,1)]*, where *S* = Source node, *D* = Sink (Destination) node, and *a*, *b*, *c*, and *d* are four nodes, and the ordered-tuple *(x, y, m, f)* denotes that for the directed edge *(x, y)* the maximum capacity is *m* but flow is *f*. Find two different maximal flows and value of each of these flows.

Marks Q3 [7.5 + 7.5 = 15]

Q4. A *contiguous subsequence* of a list S is a subsequence made up of consecutive elements of S. For instance, if S is *5*, *15*, *-30*, *10*, *-5*, *40*, *10*, then *15*, *-30*, *10* is a contiguous subsequence but *5*, *15*, *40* is not. Give a linear-time *Dynamic-Programming* algorithm (along with the recursions used and time-complexity derivation) for the following task:

Input: A list of integers, $a_1, a_2, \ldots, a_n$.

Output: The contiguous subsequence of maximum sum (and also the sum). A subsequence of length zero has sum zero.

For the preceding example, the answer would be 10, -5, 40, 10, with a sum of 55. Show the working of your algorithm for the above example.

Marks Q4 [15]