BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI,
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Open Book, Open Laptop, **No Internet**

Subject Name: CS F407 - Artificial Intelligence — Date: 13 March 2023
Examiner Name: Sravan Danda — Marks: 30
Duration: 1.5 hours

Attempt all questions. Marks corresponding to each question is highlighted in bold within square braces at the end of the question. In case of ambiguities in any of the questions, clearly state your assumptions and attempt the question(s).

1. Suppose the state space is given by the infinite integer grid $\mathbb{Z} \times \mathbb{Z}$. The initial state is $(0, 0)$ and the goal state is (n, n) for some fixed $n \in \mathbb{N}$ with $n > 5$. Further suppose that edges (valid actions) exist between states whose indices differ in exactly one of the coordinates by 1 i.e. an action at state (x_1, y_1) can only lead to the states (x_2, y_2) that satisfy $|x_1 - x_2| + |y_1 - y_2| = 1$. All the edge weights (action costs) are provided and are within the range $[1 - \epsilon, 1 + \epsilon]$ for some fixed $0 < \epsilon < \dfrac{1}{n}$. We are interested in finding an optimal path (i.e. minimizing the sum of costs of edges on the path) between the initial state and the goal state.

   (a) Does there exists a path (between the initial and goal states) of cost at most $2n + 2$? Justify.

   (b) Does any path (between the initial and goal states) that contains more than $2n + 5$ edges an optimal solution? Justify.

   (c) Using (a) and (b) or otherwise, argue that an optimal solution of the search problem exists. Also, argue that an optimal solution can be obtained by exploring a finite subset of the state space.

   (d) Suppose a depth-limited search with depth limit l is applied to obtain an approximate optimal path between the initial state and the goal state. What is the lowest possible value of l that ensures the cost C of the obtained solution with the following guarantees (C^* denotes the cost of an optimal solution)? Justify.
   $$0 \leq C - C^* \leq 4 \tag{1}$$

   [2+2+2+2]

2. Let $\mathbb{Z}_n$ denote the set of integers $\{1, 2, \cdots, n\}$ where $n \geq 100$ is a fixed positive integer. A total of n^2 vehicles denoted by V_{xy} for each $x, y \in \mathbb{Z}_n$ are to be moved from initial positions $(x, y, 1)$ to respective goal positions (y, x, n) in the least number of time steps. Two vehicles cannot occupy the same position. At each time step, exactly one of the n^2 vehicles (say V) is allowed to move as per the following rules

   - Either exactly one step to an empty neighbouring slot i.e. if the current position of V is (a, b, c) then the new position of V is (a', b', c') such that $|a - a'| + |b - b'| + |c - c'| = 1$ is satisfied and that no other vehicle is currently at (a', b', c'),
   - or hop one step over a vehicle at a neighbouring slot that differs in the same coordinate. For instance, if the neighbouring blocked position differs in the x coordinate, the hop is allowed only to a neighbour (different from the current position) that differs in the x coordinate of the blocked position. More precisely, if the current position of V is (a, b, c) and there is another vehicle at (a', b', c') with $|a-a'|+|b-b'|+|c-c'| = 1$, then the new position of V is (a'', b'', c'') with $|a'-a''|+|b'-b''|+|c'-c''| = 1$, $|(a-a'')(b-b'')|+|(c-c'')(b-b'')|+|(a-a'')(c-c'')| = 0$ and $|a - a''| + |b - b''| + |c - c''| > 0$

   where $a, a', a'', b, b', b'', c, c', c'' \in \mathbb{Z}_n$.

   (a) Provide an expression for the size of the state space as a function of n. You need not simplify the expression.

(b) Provide an expression for the maximum branching factor as a function of n.

(c) We are interested in applying the A^* search algorithm which requires construction of a non-trivial admissible heuristic. Let h_{xy} denote a generic estimate for the number of moves vehicle V_{xy} will require to get to its goal location (y, x, n) from its current location (a, b, c) with $a, b, c \in \mathbb{Z}_n$ in the presence of the remaining $n^2 - 1$ vehicles on the grid on fixed but unknown positions. For V_{xy} to reach its goal (irrespective of others reach their goal or not), is

$$h_{xy} : \mathbb{Z}_n \times \mathbb{Z}_n \times \mathbb{Z}_n \to \mathbb{R} \tag{2}$$

given by mapping (a, b, c) to $|a - y| + |b - x| + |c - n|$ an admissible heuristic? Justify.

(d) Suppose if

$$h_{xy} : \mathbb{Z}_n \times \mathbb{Z}_n \times \mathbb{Z}_n \to \mathbb{R} \tag{3}$$

is given by mapping (a, b, c) to $\dfrac{|a - y| + |b - x| + |c - n|}{2}$. Is $\displaystyle\sum_{x=1}^{n}\sum_{y=1}^{n} h_{xy}$ admissible for the original problem of moving all n^2 vehicles to their goals? Justify.

[1+2+(1+2)+(1+2)]

3. Consider the following stochastic two-player zero-sum game. Player 1 starts the game and the players take turns alternatively until the game ends with one of them winning the game and the other losing. Assume that there are two urns each of the them initially containing 2 identical objects. On each turn, a player tosses a fair coin to choose an urn from which they must remove. A player has to remove at least one object from the urn decided by the toss. A player wins if they ensure that the opponent is the first player to remove the last object from one of the urns. (Hint: The root of the game tree is a chance node. Draw the complete game tree indicating the utility values of player 1 at all the leaf nodes (i.e. terminal states). At each internal node, indicate Max, Min, chance. At each chance node indicate the current number of objects in the urns. Note that the utility value of a player winning is 1 and the utility value of a player losing is -1.).

(a) How many nodes in the complete game tree correspond to

i. chance nodes.

ii. Player 1s turn.

iii. Player 2s turn.

iv. terminal nodes

(b) Among the terminal nodes, how many correspond to player 1 winning?

(c) Consider only the terminal nodes that can be reached if both players play optimally. How many such nodes correspond to player 1 winning? Justify.

[4+1+2]

4. Suppose that a constraint satisfaction problem (CSP) has n variables each of whose domain D is identical with $|D| = d$, and consists only of binary constraints of order $\mathcal{O}(n)$. Assume that each of the binary constraints is of the form $s_1 \neq s_2$ where s_1 and s_2 are the variables. Suppose the graph of the CSP is known to have a smallest cycle cutset of size 1 i.e. there exists a node whose removal will induce a tree graph.

(a) Describe an algorithm for finding a minimal cycle cutset whose run time satisfies $\mathcal{O}(n^2)$ (You are allowed to use standard algorithms such as Breadth-First Search or Depth-First Search and their time and space complexities directly).

(b) Using (a) or otherwise describe a brief sketch of an algorithm to solve the CSP in $\mathcal{O}(nd^2 + n^2)$ (You may assume that a tree-structured CSP with binary constraints on n variables with common domain size d can be solved in $\mathcal{O}(nd^2)$ using TREE-CSP-SOLVER discussed in the lectures). Justify the time complexity of your algorithm.

[(2+1)+(1+2)]