

**Answer all parts of a question together. Start each answer from a new page of your answer sheet.**

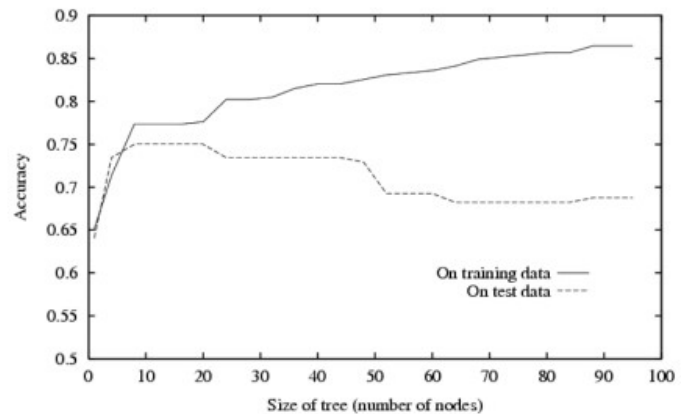
**Q1 [4M].** Assuming A, B, and C are Boolean random variables, answer the following with justification:

a) True/False:  $P(A|B)P(B) = P(B|A)P(A)$

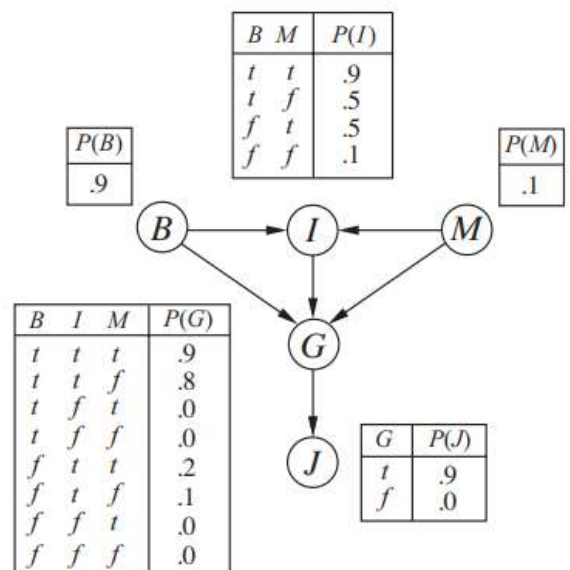
Give a two-line proof.

b) True/False:  $P(A, B, C) = P(A|C)P(B|C)P(C)$

c) Consider the plot showing training and test set accuracy for decision trees of different sizes, using the same set of training data to train each tree. Describe in one sentence how the training data curve (solid line) will change if the number of training examples approaches infinity. In a second sentence, describe what will happen to the test data curve under the same condition.



**Q2 [8M].** If a politician breaks election law (B), he might be indicted (I). His probability of being indicted is also influenced by whether or not the prosecuting lawyer is politically motivated (M). If he is indicted, he may be found guilty (G). The probability of being found guilty is also affected by whether he broke the law and whether his prosecutor is politically motivated. If he's found guilty, he has a high probability of going to jail (J). The Bayes net shown describes this process.



- a) For each of the following statements, write "True" if the statement must be true because of the network structure; else write "False". If you write "True", also write the conditional independence property which makes the statement true. If you write "False", write the conditional independence assumption that would be necessary to make it true. You should not have to compute any probabilities to answer.

(i)  $P(J | G) = P(J | G, I)$       (ii)  $P(M | G, B, I) = P(M | G, B, I, J)$

- b) Calculate the value of  $P(b, i, \neg m, g, j)$ .
- c) Calculate the probability that a politician goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.
- d) After someone has been found guilty, the president can pardon them and guarantee that they cannot go to jail (though this is quite rare). Draw the new Bayes net with P = Presidential Pardon added to the network. Introduce edges from and to P to reflect its dependence on other variables in the Bayes net. Explain what conditional independence assumptions you made and why. Then, fill in all conditional probabilities involving the new variable P with reasonable probabilities reflecting the real world.

**Q3. [3M] Answer the following:**

- a) Define node consistency and arc consistency for a CSP.
- b) True/False with justification. "All CSP's are commutative". A problem is commutative if the order of application of any given set of actions has no effect on the outcome.

**Q4 [3M].** Natural languages typically have some structure. For example, in English, it's very common for the letter 'u' to follow the letter 'q', while this is not the case in transliterated Arabic. It is required to design a Natural Language identifier which takes a text written in some Natural language as input, and outputs the most probable language. Answer the following:

- a) Is single word, dictionary based, bag-of-words a good model to design the above application. Justify.
- b) Irrespective of your answer in part (a) above, suggest a way to model the solution. Give mathematical explanation/justification wherever required.
- c) Suppose there are 100 characters in our language, and our model says that they are all equally likely. What is the perplexity for a sequence of any length?