# Birla Institute of Technology \& Science, Pilani <br> First Semester 2022-2023 <br> Graph Mining (CS F426) <br> Comprehensive Exam 2022 <br> PART A (Open Book) 

Date
: Dec 22, 2022
Duration: 1hr 45min
Total Marks : 45

## Instructions:

1. There are a total of 08 questions. All questions are compulsory.
2. Write important intermediate steps in numerical. Directly writing the final correct answer is not sufficient to obtain full marks.

Q1 [2 marks]. The unnormalized Laplacian matrix L of a simple graph holds the following properties? True/False
a) L is symmetric and positive definite.
b) The number of (linearly independent) eigenvectors with zero eigenvalues for the Laplacian matrix L is equal to the number of connected components in the underlying graph.
c) The second smallest eigenvector of Laplacian can be used to define optimal clustering of nodes into k clusters.

Q2 [1 mark]. In order to design a GNN framework for graph level tasks, which of the following layers combinations graph filtering- activation-pooling is/are possible?
a) $1-2-1$
b) $2-2-2$
c) $2-2-1$
d)None of these

Q3 [2 marks]. At graph level learning, which of the following is/are true about flat pooling?
a) It directly generates a graph representation from node representations.
b) It generates a new graph at each step until a single node graph is obtained.
c) It can be performed by adding a new node to the graph which is connected to all the nodes of the graph.

Q4 [4+2=6 marks]. For a RESCAL decoder, where $n$ and $m$ are the number of entities and relations respectively and $R_{r} \in R^{d X d}$ is a trainable matrix for each relation $r \in\{1,2, \cdots, m\}$. How many total parameters are required to learn? Also explain why RESCAL is known as a 3-way interaction model.

Q5 [4+6+2=12 marks]. For the given graph with 6 nodes, shown below

a) Compute adjacency, degree, and Laplacian matrices.
b) Let us assume that eigenvalues of the Laplacian matrix are $0,3,1,3,4$ and 5 . Compute the eigenvectors corresponding to the first two smallest eigenvalues.
c) Also, suggest the possible partitions of the graph using the second smallest eigenvector.

Q6 [6 marks]. Suppose you have a multi-relational knowledge graph with 1000 nodes and 200 relation types. You come up with a RGCN model to learn the embedding of nodes with two hidden layers having 8 and 16 neurons. For each layer, calculate the number of parameters to be learned, and the size of the associated feature maps assuming the effect of the self-node and its neighbors on the final embeddings differently.

Q7 [4+6=10 marks]. Suppose the graph is a chain of $n$ nodes as shown in Figure a.
a) Assume that the initial $h$ is a column vector of ones. Compute the final hub and authorities vectors.
b) If a self-loop is added at the first node as shown in Figure b, compute hub and authority vectors as a function of $k$, where $k$ is the number of iterations. At every step, normalize hub and authority vectors such that the maximum component of the vector is 1 .


Figure a
Figure b
Q8 $[\mathbf{3 + 2 + 1}=\mathbf{6} \mathbf{~ m a r k s}]$. In the TransE model, a triplet $(h, r, t)$ holds such that $h+r \approx t$.
a) Identify whether the given loss function will be optimized so that the valid triples are ranked above the corrupt triples or not? Justify your answer.

$$
L=\sum_{(h, r, t) \in S} \sum_{\left(h^{\prime}, r, t^{\prime}\right) \in S^{\prime}}(\lambda+d(h+r, t))+d\left(h^{\prime}+r, t^{\prime}\right)
$$

where $S$ and $S^{\prime}$ represent the set of true and corrupt triplets respectively, $d($.$) is a distance$ function and $\lambda$ is a margin constant.
b) The TransE model works well for one to one and one to many relations. True/False. Prove your answer.
c) TransE is a two-way interaction model. True/False?
$* * * * * * * * * * * * * * * * * * * * * * * * * * * * * \mathbf{E N D}=* * * * * * * * * * * * * * * * * * * * * * * * * *$

# Birla Institute of Technology \& Science, Pilani <br> First Semester 2022-2023 <br> Graph Mining (CS F426) <br> Comprehensive Exam 2022 <br> PART B (Closed Book) 

Date
: Dec 22, 2022
Duration: 1hr 15min
Total Marks : 30
Instructions:

1. There are a total of 06 questions. All questions are compulsory.
2. Write important intermediate steps in numerical. Directly writing the final correct answer is not sufficient to obtain full marks.

Q9 [4 marks]. Write down TransE and TransH decoder equations and compare their representational abilities.

Q10 [6 marks] Discuss three advantages of using graph neural network models over shallow embedding approaches to generate node embedding for solving downstream tasks such as classification, link prediction etc.

Q11 [2 marks]. Graph based learning is often referred to as semi-supervised learning. Why?

Q12 [5 marks]. The Jarvis-Patrick algorithm, unlike k-means, automatically determines how many clusters there are; it is still dependent on different input parameters. Explains? Discuss how the Jarvis-Patrick algorithm is similar to k-means, in the sense that results of the clustering are dependent on the parameters? Also, is it possible to partition both directed and undirected graphs using the Jarvis-Patrick clustering? Justify your answer.

Q13 [ $\mathbf{6 + 2}=\mathbf{8}$ marks]. Let's assume there are a total of 3204 articles from the New York times newspaper belonging to six different classes: entertainment, economy, international, national, horoscope and sports. The k-means clustering is applied and grouped these articles into 3 clusters as shown in the below table. The first column of the table indicates the cluster and the next six columns together form a confusion matrix i.e. how the articles from each category are distributed in clusters.

| Cluster | Entertainment | Economy | international | national | Horoscope | sports | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\# 1$ | 1 | 1 | 0 | 11 | 4 | 676 | 693 |
| $\# 2$ | 27 | 89 | 333 | 827 | 253 | 33 | 1562 |
| $\# 3$ | 326 | 465 | 8 | 105 | 16 | 29 | 949 |
| Total | 354 | 555 | 341 | 943 | 273 | 738 | 3204 |

Compute the purity and entropy of each cluster (using class information available in the confusion matrix) and determine which is/are the best cluster(s) in terms of entropy and purity both. Note: Use log base 2 for calculation of entropy.

Q14. [5 marks] You are asked to design a Graph Convolution Network architecture with an input layer, two hidden layers $H_{1}$ and $H_{2}$ and one output layer. Write down the graph level equation for node representation using UPDATE and AGGREGATION function at $H_{1}$ and $H_{2} . X$ and $A$ represent the input node feature and adjacency matrix respectively. Also explain the strategy to merge UPDATE and AGGREGATION steps together and limitations if any?

