Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2023-2024, MATH F421 - CS F451

Mid-semester Exam (Closed Book)

Max. Time: 50 Minutes **Date: October 14, 2023** **Max. Marks: 38**

There is **no partial marking**. Only write down the **final answer** in the provided space. Calculators are not allowed.

Name: **ID:** **Part-A**

Q.1 Write solution of the recurrence relation **[6]**

$$a_n = 7a_{n-1} - 10a_{n-2} - 2 \times 3^n;\ a_0 = 12, a_1 = 39.$$

Answer.

Q.2 Write down the generating function for the sequence $c_r = 3r + 5$. **[4]**

Answer.

Q.3 Let a_r be the number of non-negative integer solutions to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq r$$

where $3 \leq x_1 \leq 9$, $1 \leq x_2 \leq 10$, $x_3 \geq 2$ and $x_4 \geq 0$. Write the generating function for the sequence a_r in closed form. Hence, find a_{20} (leave the answer in the form of $\binom{n}{r}$). **[4+3]**

Answer.

Q.4 Write down the number of self-conjugate partitions of 15. **[4]**

Answer.

Q.5 Write the rook polynomial for the chessboard in Figure 1. **[6]**

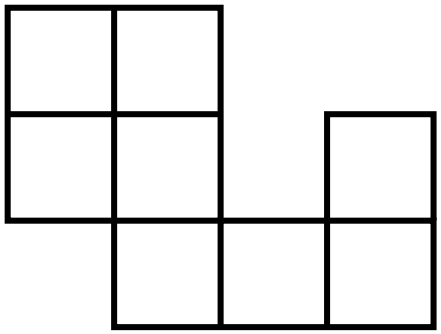

Figure 1:

Answer.

Q.6 If the polynomial $1 - x + x^2 - x^3 + \cdots + x^{16} - x^{17}$ is written in the form of $a_0 + a_1 y + a_2 y^2 + a_3 y^3 + \cdots + a_{16} y^{16} + a_{17} y^{17}$, where $y = x + 1$ and a_i's are constants, then the value of a_2 is $\binom{n}{r}$. Find n and r. **[6]**

Answer.

Q.7 A student must answer 5 out of 10 questions on a test. The five questions that are answered must include at least 2 of the first 5 questions. If the order of the answers is not important, in how many different ways can this be done (write final answer as a natural number)? **[5]**

Answer.