

Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2023-2024, MATH F421 - CS F451

Comprehensive Examination (Part B: Open Book)

Time: 80 Minutes

Date: December 20, 2023

Max. Marks: 40

Calculators are not allowed.

PART-B

- Q.1** Using Burnside Lemma, compute, how many ways are there to put 16 identical balls in four identical boxes at the four vertices (one box at each vertex) of a square board, allowing empty boxes, assuming that the board can freely rotate? [9]
- Q.2** Using Polya's theory, find, how many different ways are there to color the vertices of a pyramid (that is free to move in space) with white and blue such that three vertices are white and two are blue? [9]
- Q.3** Let $a_n = 2^n - 1$, $n \geq 1$ and m be an arbitrary odd positive integer. Using Pigeonhole principle, prove that m divides at least one of a_1, a_2, \dots, a_m . [9]
- Q.4** Compute $r(C_4, C_4)$, where C_4 is a cycle of length 4. Justify all the cases by drawing graphs. [13]