Birla Institute of Technology and Science - Pilani, KK Birla Goa Campus
First Semester, 2019-2020
Dynamics and Vibrations (DE G611)

Date: 6th December 2019, Time: 2.00 PM: 05.00 PM, Total Marks: 70

1.

i. If the characteristic roots have zero imaginary part for an SDOF system, the response of the system will be
   a) oscillatory b) non-oscillatory c) steady d) Periodic

ii. The force at point '*i*' due to a unit displacement at point '*j*', when all the points other than the point '*j*' are fixed, is known as __________________influence coefficient

iii. The stiffness matrix of an unstable system is ___________________________ (Positive definite/ Semi-definite / Negative definite)

iv. The stiffness matrix of the system shown in Question No. 8 is __________________ (Positive definite/ Semi-definite / Negative definite)

v. An automobile having a mass of 2,000 kg deflects its suspension springs 0.02 m under static conditions. The natural frequency of the automobile is _____________________

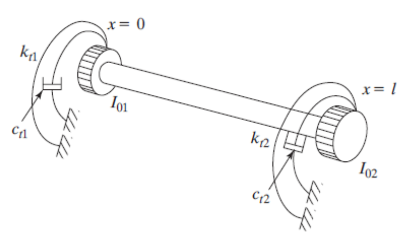
(05 Marks)

2. The needle indicator of an electronic instrument is connected to a torsional viscous damper and a torsional spring. If the rotary inertia of the needle indicator about its pivot point is 25 kg-m^2 and the spring constant of the torsional spring is 100 N-m/rad, determine the damping constant of the torsional damper if the instrument is to be critically damped.

(05 Marks)

3. Show that the determination of fundamental natural frequency of an MDOF system using Rayleigh's method results only in second-order errors even if the assumed mode of vibration contains first-order errors **(10 Marks)**

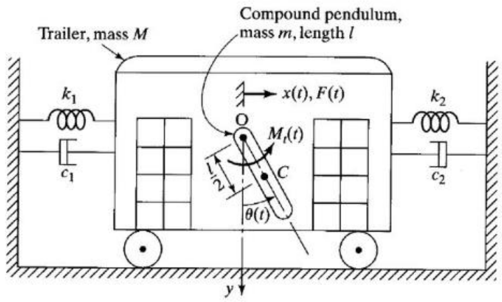
4. A uniform shaft of length *l* and torsional stiffness *GJ* is connected at both ends by torsional springs, torsional dampers, and discs with inertias, as shown in Fig. State the boundary conditions.



(10 Marks)

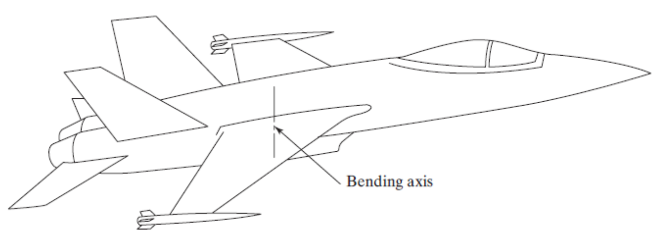
5. Write down the equations for kinetic and potential energy for the system shown in the figure below(Assume $c_1 = c_2 = 0$)

(10 Marks)



6. The wing of a fighter aircraft, carrying a missile at its tip, as shown in Fig., can be approximated as an equivalent cantilever beam with $EI = 15X10^9 N-m^2$ about the vertical axis and length *l=10m*. If the equivalent mass of the wing, including the mass of the missile and its carriage system, at the tip of the wing is *m=2500kg*, determine the vibration response of the wing (of m) due to the release of the missile. Assume that the force on the tip due to the release of the missile can be approximated as an impulse function of magnitude *F = 50 N-s*. The mass of the missile is negligible. *(Hint: The system can be modelled as an SDOF system; The tip deflection of a cantilever beam can be expressed as* $\delta_{max} = \frac{Wl^3}{3EI}$ *)*

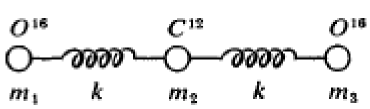
(10 Marks)



(a) Real system

7. Derive an expression for finding the longitudinal vibration response of a bar using Lagrangian approach. **(10 Marks)**
8. The CO_2 molecule can be likened to a system made up of a central mass m_2 connected by equal springs of spring constant *k* to two masses m_1 and m_3 as shown in the Fig. Write down the equations of motion for the above system

. **(10 Marks)**



Solutions:

1	i. Non- oscillatory ii. Stiffness influence coefficient iii. Negative definite iv. Semi-definite v. 22.14rad/s
2	Equation of motion : $J_0 \ddot{\theta} + C_t \dot{\theta} + k_t \theta = 0$ with $J_0 = 25$ Kg-m^2 and $k_t = 100$ N-m/rad. For critical damping, Eq. (2.105) gives $c = C_c = 2\sqrt{J_0 k_t} = 2\sqrt{25(100)}$ $= 100$ N-m-s/rad.
3	Ref: Page 659 SS Rao (Edition 6)
4	Boundary conditions: At $x = 0$, $GJ \frac{\partial \theta}{\partial x}(0,t) = k_{t1}\,\theta(0,t) + c_{t1}\frac{\partial \theta}{\partial t}(0,t) + J_1 \frac{\partial^2 \theta}{\partial t^2}$ At $x = l$, $GJ \frac{\partial \theta}{\partial x}(l,t) = -k_{t2}\,\theta(l,t) - c_{t2}\frac{\partial \theta}{\partial t}(l,t) - J_2 \frac{\partial^2 \theta}{\partial t^2}$
5	$r_g = \left(x + \frac{l}{2}\sin\theta\right)i + \left(-\frac{l}{2}\cos\theta\right)j$ $v_G = \dot{x}\hat{I} + \frac{l\dot{\theta}}{2}\hat{j} = (\dot{x} + \frac{l\dot{\theta}}{2}cos\theta)\hat{I} + (\frac{l\dot{\theta}}{2}sin\theta)\hat{J}$ The square of its magnitude is given by $v_G^2 = (\dot{x} + \frac{l\dot{\theta}}{2}cos\theta)^2 + (\frac{l\dot{\theta}}{2}sin\theta)^2$ $v_G^2 = \dot{x}^2 + 2\frac{l\dot{x}\dot{\theta}}{2}cos\theta + (\frac{l\dot{\theta}}{2})^2cos^2\theta + (\frac{l\dot{\theta}}{2})^2sin^2\theta$ $v_G^2 = \dot{x}^2 + \dot{x}\dot{\theta}lcos\theta + \frac{l^2\dot{\theta}^2}{4}$

The pendulum is exhibiting planar motion (translation and rotation), therefore its KE will be

$$T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega^2$$

Kinetic Energy

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2 {v_G}^2 + \frac{1}{2}I_G\dot{\theta}^2$$

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + \dot{x}\dot{\theta}l\cos\theta + \frac{l^2\dot{\theta}^2}{4}) + \frac{1}{2}(\frac{ml^2}{12})\dot{\theta}^2$$

Potential Energy

$$V = \frac{1}{2}kx^2 + m_2 g\frac{l}{2}(1 - \cos\theta)$$

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(4.38) The stiffness of the cantilever beam (wing) is given by

$$k = \frac{3EI}{l^3} = \frac{3\,(15 \times 10^9)}{10^3} = 45 \times 10^6 \ \text{N/m}$$

System can be modeled as a single degree of freedom undamped system:

$$m\ddot{x} + kx = 0$$

where $m = 2500$ kg, $k = 45 \times 10^6$ N/m, and

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{45 \times 10^6}{2.5 \times 10^3}} = 134.1641 \ \text{rad/s}$$

Response of mass due to impulse $\underset{\sim}{F}$ is given by Eq. (4.26) with $\zeta = 0$ and $\omega_d = \omega_n$:

$$x(t) = \frac{\underset{\sim}{F}}{m\,\omega_n} \sin\omega_n t$$

$$= \frac{50}{2500\,(134.1641)} \sin 134.1641\,t$$

$$= 0.000149071 \sin 134.1641\,t \ \ \text{m}$$

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In the case of longitudinal vibration of a bar, the kinetic energy is given by

$$\mathcal{T} = \frac{1}{2}\int_0^l \rho A u_{,t}^2 \,\mathrm{d}x.$$

$$\mathcal{V} = \frac{1}{2}\int_0^l \sigma_x \epsilon_x A \,\mathrm{d}x = \frac{1}{2}\int_0^l EA\epsilon_x^2 \,\mathrm{d}x$$
$$= \frac{1}{2}\int_0^l EAu_{,x}^2 \,\mathrm{d}x.$$

Writing the Lagrangian $\mathcal{L} = \mathcal{T} - \mathcal{V}$, Hamilton's principle assumes the form

$$\delta\int_{t_1}^{t_2} \mathcal{L}\,\mathrm{d}t = 0,$$

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$$\delta \int_{t_1}^{t_2} \frac{1}{2} \int_0^l \left(\rho A u_{,t}^2 - E A u_{,x}^2\right) \mathrm{d}x \, \mathrm{d}t = 0,$$

$$\Rightarrow \int_0^l \rho A \delta u \Big|_{t_1}^{t_2} \mathrm{d}x - \int_{t_1}^{t_2} E A u_{,x} \delta u \Big|_0^l \mathrm{d}t$$

$$- \int_{t_1}^{t_2} \int_0^l [\rho A u_{,tt} - (E A u_{,x})_{,x}] \, \delta u \, \mathrm{d}x \, \mathrm{d}t = 0.$$

$$\rho A u_{,tt} - (E A u_{,x})_{,x} = 0,$$

$$E A u_{,x}(0, t) \equiv 0 \quad \text{or} \quad u(0, t) \equiv 0,$$

$$E A u_{,x}(l, t) \equiv 0 \quad \text{or} \quad u(l, t) \equiv 0.$$

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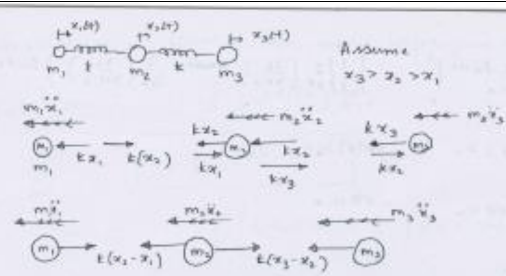
Q8)



Assume
$x_3 > x_2 > x_1$

For m_1

$$m_1\ddot{x}_1 = k(x_2 - x_1)$$

$$m_1\ddot{x}_1 + kx_1 - kx_2 = 0 \rightarrow (1)$$

For m_2

$$m_2\ddot{x}_2 = -k(x_2 - x_1) + k(x_3 - x_2) = kx_1 \mp 2kx_2 + kx_3$$

$$m_2\ddot{x}_2 - kx_1 + 2kx_2 - kx_3 = 0 \rightarrow (2)$$

For m_3

$$m_3\ddot{x}_3 = -k(x_3 - x_2) = -kx_3 + kx_2$$

$$m_3\ddot{x}_3 - kx_2 + kx_3 = 0 \rightarrow (3)$$

eq^{n}s (1), (2) & (3) can be written in matrix form as

$$\boxed{\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}}$$ Ans

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