Birla Institute of Technology and Science, Pilani

Comprehensive Examination: ECE / EEE / INSTR F243: Signals and Systems

Marks: 100 **AY: 2022-23, Semester: II** **Date: 20-May-2023, Saturday**

Time: 180 minutes **CLOSED BOOK** **Pages: 02**

Q1	A causal LTI system is described by the difference equation $y[n] = y[n-1] + 0.5y[n-2] + x[n] + x[n-1]$, obtain a) Impulse response $h[n]$, and sketch pole-zero plot of $H(z)$ b) Step response $s[n]$, and sketch pole-zero plot of $S(z)$ [8+7]	[15]
Q2	The signal $y(t) = e^{-2t}u(t)$ is the output of a causal all-pass system for which the system function is $H(s) = \frac{s-1}{s+1}$ (a) Find two possible inputs $x(t)$ that could produce $y(t)$ and sketch respective ROC's (b) What is the input $x(t)$ if it is known that $\int_{-\infty}^{\infty} \lvert x(t) \rvert dt < \infty$ (c) Find a stable (but not necessarily causal) impulse response $h_1(t)$ of a system that has $x(t)$ as an output and $y(t)$ is the input. [8+2+5]	[15]
Q3	An LTI system has input $x(t)$, output $y(t)$, and impulse response $h(t)$. If the step response of a system is $s(t) = \{u(2+t) - u(t-2)\} * \{u(t+2)u(2-t)\}$, and input $x(t) = \frac{d^2}{dt^2}\left\{s\left(\frac{t}{2}\right)\right\}$, compute and sketch the system output $y(t)$, where * represents linear convolution.	[15]
Q4	A periodic signal $x[n]$ has period 8 and Fourier coefficients a_k. Sketch one period of $x[n]$, with the help of the following information i. $a_k = -a_{k-4}$. ii. $x[2n+1] = (-1)^n$. Now, if a periodic signal $y[n]$ with period 8 and Fourier coefficients b_k is represented as below, obtain b_k in terms of a_k, and sketch one period of $y[n]$ $y[n] = \left(\frac{1+(-1)^n}{2}\right)x[n-1]$ [8+7]	[15]

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Q5 A discrete-time LTI system has input $x[n]$, impulse response $h[n]$, and output $y[n]$. [20]

Impulse response h[n] and input x[n] are represented below

$$h[n] = 8(-1)^n \left(\frac{\sin\left(\frac{\pi n}{8}\right)}{\pi n} \cdot \frac{\sin\left(\frac{3\pi n}{8}\right)}{\pi n} \right)$$

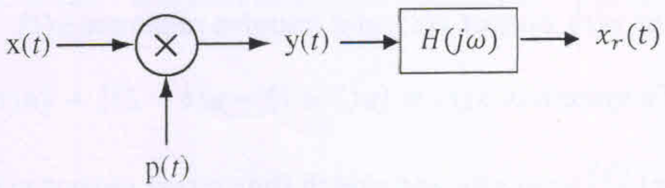
$$x[n] = \frac{16}{\pi} x_1[n] \cos\left(\frac{3\pi n}{8}\right) + 2x_2[n] \cos\left(\frac{7\pi n}{8}\right)$$

where

$$x_1[n] \overset{DTFT}{\longleftrightarrow} X_1(e^{j\omega}) = \begin{cases} j\omega & |\omega| < \frac{\pi}{8} \\ 0 & |\omega| > \frac{\pi}{8} \end{cases} \qquad x_2[n] \overset{DTFT}{\longleftrightarrow} X_2(e^{j\omega}) = \begin{cases} \cos(4\omega) & |\omega| < \frac{\pi}{8} \\ 0 & |\omega| > \frac{\pi}{8} \end{cases}$$

Sketch $|X(e^{j\omega})|$, $|H(e^{j\omega})|$, and $|Y(e^{j\omega})|$ for the interval $-2\pi \le \omega \le 2\pi$. [10+5+5]

Q6 A sampling process is represented by the figure below: [20]



$x(t) = (\cos(100t))^2$ is an input signal, $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ is an impulse train, n is an integer, T_s is sampling period, $h(t) = \left(\frac{\sin 200t}{\pi t}\right)^2$ is the impulse response of a filter.

a. Sketch the magnitude response of $X(j\omega)$ and $Y(j\omega)$, if the sampling frequency is 25% above the Nyquist rate.

b. If the sampling frequency is 25% below the Nyquist rate, sketch the magnitude response of $X_r(j\omega)$ and $H(j\omega)$.
Represent the reconstructed signal $x_r(t)$. [10+10]