Birla Institute of Technology and Science, Pilani

Digital Signal Processing (EEE/ECE F434)

Mid-Semester Examination, First Semester 2023 – 24

Time: 90 minutes | Maximum Marks: 60

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1. Answer the following with justifications [no marks, if no justification].

   (a) During the live telecast of the successful Chandrayaan-3 mission soft landing, consider that the digital audio signal received was $x(n)$ with DTFT $X(w)$. As a DSP engineer, explain which signal had more important perceptual information about the received audio signal: the DTFT magnitude $|X(w)|$ or phase $\angle X(w)$. **(3)**

   (b) Prove the following property as true or false **(3)**

$$\frac{1}{N}\left[1 + W_N^k + W_N^{2k} + ... + W_N^{(N-1)k}\right] = \delta(k),\ k = 0, 1, ..., N-1.$$

   (c) During an important product release a DSP engineer has to implement an inverse DFT, but the hardware device available is capable of computing only a direct DFT. Show how to use the direct DFT to perform an inverse DFT. **(3)**

   (d) In the lecture, we discussed how to perform linear convolution of sequences of lengths N_1, N_2 by zero padding to length $N_1 + N_2 - 1$. Suppose, instead, that we zero pad the two sequences to length $M > N_1 + N_2 - 1$. How then is the circular convolution of the zero-padded sequences related to the linear convolution? **(3)**

   (e) A system H has an impulse response given by $h[n] = (\underline{0.5}, 0, 0.5)$, where the underline indicates the origin. What kind of system does this represent? Find and sketch the magnitude and phase response of this system. **(5)**

2. Let us define a new transform $X^g[k]$ of a real sequence $x[n], 0 \leq n \leq N-1$, as

$$X^g[k] = \sum_{n-0}^{N-1} x[n]\, W_N^{-k(n+0.5)}$$

   (a) Compute a relation between $X^g[k]$ and DFT $X(k)$. **(2)**

   (b) Let $y[n]$ be a sequence such that

$$y[n] = \begin{cases} x[n], & 0 \leq n \leq N-1, \\ x[2N-n-1], & N \leq n \leq 2N-1. \end{cases}$$

   Compute $Y^g[k]$ and express it as a function of $X(k)$ for even values of k. Is the result always real or complex in general? **(4)**

3. We have a DT signal, $x[n]$, arriving from a source at a rate of $1/T_1$ samples per second. We want to digitally resample it to create a signal $y[n]$ that has $1/T_2$ samples per second, where $T_2 = \frac{3}{5}T_1$

   (a) Draw a block diagram of a DT system to perform the resampling. Specify the input/output relationship for all the boxes in the Fourier domain. **(5)**

   (b) For an input signal $x[n] = \delta[n]$, determine $y[n]$. **(4)**

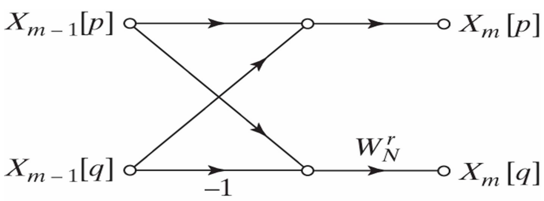

Figure 1: DIF radix-2 FFT butterfly.

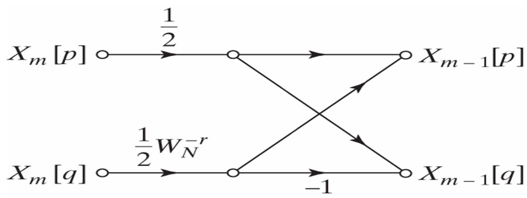

Figure 2: Proposed FFT butterfly.

4. In the lecture, we had discussed that the basic butterfly flow-graph of the DIF radix-2 FFT algorithm is as shown in Figure 1.

   (a) Show that $X_{m-1}[p]$ and $X_{m-1}[q]$ can be computed from $X_m[p]$ and $X_m[q]$, respectively, using the butterfly shown in Figure 2. **(4)**

   (b) If you recall the lecture, in the DIF algorithm, the input sequence is arranged in the normal order and the DFT is arranged in the bit-reversed order [Figure 3 is given as a reference]. If each butterfly is replaced by the appropriate butterfly of the form of Figure 2, the result would be a flow graph for computing the sequence $x[n]$ in normal order from the DFT $X[k]$ in the bit-reversed order. Draw the resulting flow graph for $N = 8$. **(8)**

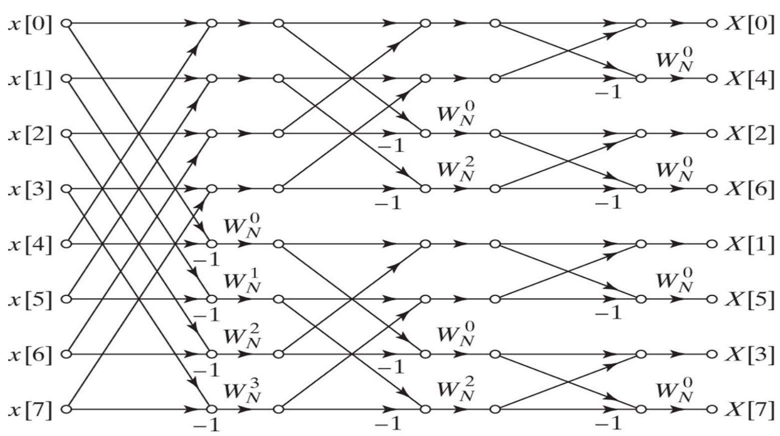

Figure 3: DIF butterfly discussed in lecture, in case you do not remember.

   (c) If you observe carefully, the flow graph you have obtained in part (b) is for computing inverse DFT. Modify the graph so that it computes DFT instead of inverse DFT. **(6)**

   (d) Observe that the result in part (c) is the transpose of the DIF algorithm and that it is identical to the DIT algorithm, as discussed in the lecture. Does it follow that, to each DIT algorithm, there corresponds a DIF algorithm that is the transpose of the DIT algorithm and vice versa? Explain. **(2)**

5. A LTI system with input $x[n]$ and output $y[n]$ satisfies the difference equation

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n] - x[n-1].$$

   Find all the possible values for the system's impulse response $h[n]$ at $n = 0$. Also, give the expression of $h[n]$ for each case. **(8)**