Birla Institute of Technology and Science, Pilani

Digital Signal Processing (ECE/EEE F434)

Comprehensive Examination, Closed Book, First Semester 2023 – 24

Maximum Time: 120 minutes Date: 12/12/23 Maximum Marks: 50

1. Justify the following with reasoning, no marks, if no justification.

   [*Reasoning/Justification: brevity is the key, it saves paper and time*]

   (a) During the recently held cricket world cup final match, your friend who happens to be an audio enthusiast used an analog audio system as he/she believes that analog audio representation of the original sound source is more exact than in a digital audio system. Is this true or false? **(2)**

   (b) Linear convolution between the channel input, $x(n)$, and impulse response, $h(n)$, can be turned into a circular convolution by adding a cyclic prefix, prove the correctness of this statement in the context of an OFDM-based wireless communication system. Justify mathematically. **(3)**

   (c) "Impulse response $h(n) = \delta(n) - \delta(n-6)$, represents a Comb filter". If the given statement is correct, what is the order of the filter, if incorrect, what is the system represented by the given impulse response. **(2)**

   (d) Examine the following claim: "Every real periodic DT signal is necessarily a finite sum of DT sinusoidal signals." If the claim is right, explain why. If it is wrong, give a counterexample. **(2)**

   (e) Suppose you are told that an $N = 32$ FFT algorithm has a twiddle factor of W_{32}^{2} for one of the butterflies in its fifth stage. Is the FFT a decimation-in-time or decimation-in-frequency algorithm? Give reasons for your choice. **(2)**

2. Let's assume that you have performed a 20-point DFT on a sequence of real valued time-domain samples, and you want to send DFT results to your friend by e-mail/WhatsApp. What is the absolute minimum number of (complex) frequency-domain sample values you will need to type in your e-mail/WhatsApp message so that your friend has complete information regarding the DFT results. **(2)**

3. You are given a digital filter

$$H(z) = \frac{2\,\theta^2(1+z)^2}{(z-1)^2 + 2\,\theta(z^2-1) + 2\,\theta^2(z+1)^2},$$

   where θ is a given positive parameter. Further, $H(z)$ was obtained from an analog filter $H(s)$ by BLT technique with $T = 2$.

   (a) What type of filter is $H(z)$: LPF, HPF, BPF, or BSF? Give reasons. **(3)**

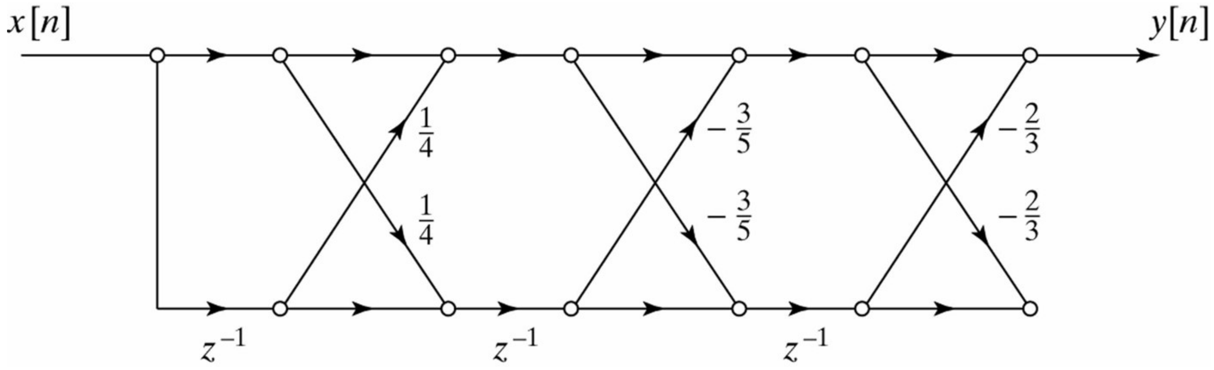
   (b) Compute the transfer function, poles, and zeros, of the analog filter $H(s)$. Is this Butterworth or Chebyshev-I filter? **(2+1+1+1)**

4. (a) One of the advantages of an FIR filter is linear phase response. What characteristic must the coefficients of an FIR filter (and the window for FIR filter design) have to ensure that its frequency-domain phase response is a linear function of frequency (i.e., linear phase). **(2)**

(b) The impulse response of a linear phase FIR filter starts at the following values: $h[0] = 1, h[1] = 3, h[2] = -2$. For type II and III FIR filter, find the coefficients of the smallest order FIR filter that satisfies this condition. **(4)**

5. (a) Determine the system function $H(z)$ relating the input $x(n)$ to output $y(n)$ for the FIR lattice structure given below. **(5)**

   (b) Give the lattice filter structure for the all pole filter $1/H(z)$. **(3)**



6. Design an IIR Butterworth digital filter to meet the following design specifications: passband gain between 0 dB and -1 dB with passband edge frequency $0.2\,\pi$ rad/sec, stop band attenuation of atleast -15 dB with stop band edge frequency of $0.3\,\pi$ rad/sec. Find the transfer function of the analog filter. Use bilinear transformation to find the digital filter transfer function. **(8)**

   Hint: *In general the analog filter TF for one of the cases is given as below, the other case can be deciphered from this, as discussed in the lecture.*

$$H(s) = \frac{\Omega_c^N}{(s+\Omega_c)\prod_{k=1}^{(N-1)/2}\left(s^2 + b_k\,\Omega_c\,s + {\Omega_c}^2\right)}.$$

7. The input to a system with a digital filter characterized by $y[n] = 0.89\,y[n-1] + x[n]$ is quantized to 8 bits. What is the power produced by the quantization noise at the output of the filter. **(3)**

8. Three students in a class namely, X, Y, and Z are given a causal and stable LTI system and are told that its transfer function is

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}.$$

   They are asked to find the parameters b_0, b_1, a_1. X feeds a unit impulse to the system and reports that $h[0] = 1$. Y feeds a unit step to the system and reports that the DC gain is 4. Z feeds a signal $\cos(\pi\, n/3)$ to the system and reports that the amplitude of the sinusoidal signal at the output is 2. Based on this information, what are the values of b_0, b_1, a_1? **(4)**