BITS PILANI K.K. BIRLA GOA CAMPUS

I SEMESTER 2022-23 COMPREHENSIVE EXAM

ECON F213: Mathematical and Statistical Methods

27/12/22 Time: 3 Hours Total Marks: 40

Instructions:

1. **If you need to make any assumptions, clearly state them**
2. **Final answers must be clearly stated along with all supporting analysis**

1. Consider a perfectly competitive coffee producing firm operating in an underdeveloped country. The profit maximizing firm produces coffee using production function $q = F(x_1, x_2)\ where\ F_1 > 0 > F_{11}$ and $F_2 > 0 > F_{22}$. Assume that output produced q depends on two variables: Input1 (x_1) and number of child labor (x_2). Each unit of Input1 is hired at the prevailing market price (denoted by w_1) and each child labor is hired at prevailing wage rate (denoted by w_2). The firm's profit maximizing choice of inputs are denoted by x_1^*, x_2^*. The objective of this exercise is to analyze how to *reduce* the use of child labor in coffee production. Consider the following predictions from three economists.

   A. Economist1 assumes that $F_{12} > 0$ and predicts that an increase in P, the price per unit of coffee would lead to decrease in use of x_2^*, ceteris paribus. Use appropriate comparative static analysis to examine this prediction. Clearly state whether you Agree/ Disagree with the prediction.

   B. Economist2 assumes that $F_{12} = 0$ and predicts that an increase in w_1 would lead to decrease in use of x_2^*, ceteris paribus. Use appropriate comparative static analysis to examine this prediction. Clearly state whether you Agree/ Disagree with the prediction.

   C. Economist3 is not sure about the sign of F_{12} but still predicts that a decrease in w_2 would lead to decrease in use of x_2^*, ceteris paribus. Use appropriate comparative static analysis to examine this prediction. Clearly state whether you Agree/ Disagree with the prediction. [5X3=15]

2. The demand functions for fish (F) and chicken (C) are specified as follows:

$$D_F = f(P_F, P_C) where\ f_1 = \frac{\partial f}{\partial P_F} < 0, f_2 = \frac{\partial f}{\partial P_C} > 0$$

$$D_C = g(P_F, P_C) where\ g_1 = \frac{\partial g}{\partial P_F} > 0, g_2 = \frac{\partial g}{\partial P_C} < 0$$

Here P_F, P_C denotes the price of fish and price of chicken respectively.

The supply of fish depends on the number of fishermen (N) and its price according to the function:

$$S_F = h(P_F, N),\ where\ h_1 = \frac{\partial h}{\partial P_F} > 0, h_2 = \frac{\partial h}{\partial N} > 0$$

The supply of chicken depends only on its price:

$$S_C = r(P_C), where\ r' > 0$$

Assume, as discussed in class, that "own price effect" dominates the "cross price effect" for both the commodities.

A. Derive the comparative static impact of a *decrease* in number of fishermen (N) on the equilibrium quantity and price of chicken. Write your FINAL ANSWER in terms of $f_i, g_{i,} h_i, r'$ [6]

B. Derive the comparative static impact of a *decrease* in number of fishermen (N) on the equilibrium quantity and price of fish. Write your FINAL ANSWER in terms of $f_i, g_{i,} h_i, r'$ [4]

3. Following is an inter-industry input output table. Cell Xij represents the *value* of Industry i output used as input in the production of Industry j output. The values are in millions of Rupees.

		To			
		Industry I	Industry II	Industry III	Consumption Demand
From	Industry I	20	60	10	50
	Industry II	50	10	80	10
	Industry III	40	30	20	40

A. Assume that Labor is the only *non-industrial* input is used in the production of each of these industrial products. Calculate $\frac{Value\ of\ Labor}{Value\ of Output}$ coefficient for each of the 3 industries. [3]

B. Suppose the Consumption Demand changes to $\begin{bmatrix}70\\25\\50\end{bmatrix}$. On the basis of the available information, calculate the new Xij and complete the following table. [9]

		To		
		Industry I	Industry II	Industry III
From	Industry I			
	Industry II			
	Industry III			

C. Using the results obtained in A and B, calculate the Total wage that must be paid to labor in order to produce the Total Output as obtained in part B. [3]