## BIRLA INSTITUTE OF TECHNOLOGY \& SCIENCE, PILANI <br> FIRST SEMESTER 2023-2024 <br> Comprehensive Examination Part-A \& B (Closed Book)

Course No.
Course Title : Mathematical \& Statistical Methods Date : 9/Dec/2023

Maximum Marks : 40
Duration (Max)
Weightage
: 90 Minutes : 20\%

## Instructions:

Write your Name and ID Number clearly on the answer sheet. There are two parts: Part A \& B; Part A carries 16 multiple-choice questions and Part B contains four short answer-type questions. Answers to questions in Parts A \& B should only be answered in the given space on this sheet; There will be no negative marking. Do not overwrite. Use of pencil is not allowed. Do not use a red color pen. A calculator is allowed; however, the exchange of a calculator is not permitted.

| PART A: Space to answer MCQs |  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Question | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| Answer |  |  |  |  |  |  |  |  |  |  |
| Question | 11 | 12 | 13 | 14 | 15 | 16 | Part A - <br> Marks | Part B - <br> Marks | Part A \& B - <br> Total Marks |  |
| Answer |  |  |  |  |  |  |  |  |  |  |

Part A: Multiple Choice Questions (1 Mark Each)
Q1: The mean of 50 numbers is zero, at most, how many may be greater than zero?
a) 49
b) 25
c) Both
(a) and (b)
d) None of the above

Q2: Find the C1, B2 and C3 in the following table: The answer mentioned in the options is in the form (C1, B2, C3)

|  | A | B | C | Combined |
| ---: | :---: | :---: | :---: | :---: |
| Number | 10 | 8 | C1 | 24 |
| Mean | 20 | B2 | 6 | 15 |
| Geometric Mean | 10 | 7 | C3 | 8.397 |

a) $6,15.5,10$
b) $10,4,15.5$
c) $6,15.5,7.997$
d) $6,24,18$

Q3: Consider a standard normally distributed variable, a t-distributed variable with $d$ degrees of freedom, and $F$-distributed variable with $(1, \mathrm{~d})$ degrees of freedom. Which of the following statement is FALSE?
a) The standard normal is a special case of the $t$ - t -distribution, the square of which is a special case of the F -distribution
b) Since the three distributions are related, the $5 \%$ critical values from each will be the same
c) Asymptotically, a given test is conducted using any of the three distributions will lead to the same conclusion
d) The normal and t -distribution are symmetric about zero while the F - takes only positive values

Q4: When the distribution is of open-end classes, which average may be appropriate?
a) Mean
b) Median
c) Mode
d) None

Q5: In a random sample of 1000 students from BITS Pilani, 510 were consumers of veg cheeseburgers. In another sample of 800 students from BITS Hyderabad, 480 were consumers of cheeseburgers. Does the data reveal a significant difference between BITS Pilani and BITS Hyderabad so far as the proportion of consumers of veg cheeseburger? [S.E. $=0.024$ ]
a) No
b) Yes, but the difference is not more than 2.58 Standard error (S.E.) at $1 \%$ level of the significance
c) Yes, the difference is more than 2.58 Standard error (S.E.) at $1 \%$ level of the significance
d) Yes, but the difference is not more than 1.96 Standard error (S.E.) at $5 \%$ level of the significance

Q6: The mean height obtained from a random sample of size 100 is 64 inches. The standard deviation of the distribution of height of the population is known to be 3 inches. The mean height of the population is 67 inches at $5 \%$ level of significance. Set up $99 \%$ limits of the mean height of the population. [S.E. $=0.3$ ]
a) 63.2 to 64.8
b) 60 to 61.7
c) 64.5 to 66.5
d) 49.2 to 53.5

Q7: Which of the following is the property of $t$-distribution?
a) The variable t-distribution can only range from 0 to plus infinity
b) The variable $t$-distribution can range from minus infinity to plus infinity
c) The variance of $t$-distribution is always less than one
d) Unlike standard normal distribution, t-distribution is unsymmetrical and has a non-zero mean

Q8: Paes, a star tennis player, is playing with the number one player in the world, Bhupathi. Before the match, Paes decided that he would serve 20 percent of his serves to Bhupathi's backhand, 30 percent of his serves to Bhupathi's forehand, and 50 percent of his serves straight at Bhupathi. In the language of game theory, this is known as:
a) a pure strategy
b) a dominant strategy
c) a mixed strategy
d) a maximin strategy

Q9: Given the utility function: $U=X^{2} \log (Y / X)$; what would be the final result of Euler's theorem?
a) 3 U
b) U
c) 2 U
d) None of the above

Q 10: Given the market supply curve $P=2+Q$. Where $P$ is price and $Q$ is quantity supplied. What would be the producer surplus when price is equal to 4 .
a) 4
b) 6
c) 2
d) None of these

Q11: For infinitely repeated games in which the players follow a tit-for-tat strategy, which one of the following outcomes is NOT possible?
a) The players cooperate until someone decides not to cooperate, and then the other players will not cooperate for some time
b) If the information about another player's action is limited, then some cooperative actions may be incorrectly interpreted as "not cooperate"
c) There can be dominant strategies
d) All of the above are possible outcomes

Q12: Consider the following game that represents the payoffs from different advertising campaigns (low, medium, and high spending) for two political candidates that are running for a particular office. The values in the payoff matrix represent the share of the popular vote earned by each candidate. Under the following version of the game with simultaneous moves, what is the Nash equilibrium?

|  | Candidate B - low | Candidate B - medium | Candidate B - high |
| :--- | :---: | :---: | :---: |
| Candidate A - low | 50,50 | 40,60 | 20,80 |
| Candidate A - medium | 60,40 | 50,50 | 35,65 |
| Candidate A - high | 80,20 | 65,35 | 50,50 |

a) Neither candidate has a dominant strategy, but the Nash equilibrium occurs where both candidates use medium advertising campaigns
b) Candidate A's dominant strategy is high, Candidate B's dominant strategy is high, and this is the Nash equilibrium
c) Neither candidate has a dominant strategy, but the Nash equilibrium occurs where both candidates use high advertising campaigns
d) This simultaneous game has no Nash equilibrium (in pure strategies)

Q 13: "If factors are paid according to their marginal productivity, total product exhaust." Given this statement, what would be the final value of Euler's theorem for the production function $Q=4 L^{\frac{3}{4}} K^{\frac{1}{4}}$.
a) $Q$
b) $4 Q$
c) $3 Q$
d) $(3 / 4) Q$

Q14: An unbounded solution in a LPP means that:
a) It has no limit on the constraints
b) The feasible region is not bounded
c) The primary variables may take any value in the unbounded region
d) All of these

Q15: Given the values of simple correlation coefficients: $r_{12}=0.9, r_{13}=0.75, r_{23}=0.7$, what would be the value of partial correlation coefficient between variables 1 and 2 , keeping the effect of 3 rd variable as constant?
a) 0.5
b) 0.9
c) 0.79
d) 0.9

Q16: A firm produces three products ( $\mathrm{A}, \mathrm{B}$, and C ), and sell in two markets ( X and Y ) at a per-unit price of INR $2,1.5$, and 1.25 , respectively. Following is the data given on the annual sales volume of three products in two markets, calculate the total revenue a firm earns.

| Market |  |
| :---: | :---: |
| $\mathbf{X}$ | $\mathbf{A}$ |
| $\mathbf{Y}$ | 4000 |
|  | 12000 |


| Products |  |
| :---: | ---: |
| $\mathbf{B}$ | $\mathbf{C}$ |
| 18000 | 20000 |
| 5000 | 3000 |

a) In Market $X=42000$, In Market $Y=20000 \quad$ b) In Market $X=60000$, In Market $Y=35250$
c) In Market $X=35250$, In Market $Y=60000$
d) In Market $X=20000$, In Market $Y=42000$

## Part B: Short Answer Type Questions

Q17: The demand for Dosa in the All Night Canteen (ANC) is $Q=360-15 P$. Four stalls sell Dosa in the ANC. Their marginal cost equals the average cost and is identical at ₹ 20 . The stall with the lower Dosa price will receive all the business, as there is no student loyalty in the ANC. If all the stalls set the same price, they will split the business equally. a) If all the stalls engage in price competition in a oneperiod game, what is the equilibrium price and quantity of Dosa sold in each stall in the ANC? b) If the four stalls agree to collude and the market demand remains the same, what is the equilibrium price and quantity of Dosa sold per stall in the ANC?
[4M]
Q18: With the help of an input-output table for an economy of three sectors, Agriculture, Manufacturing, and services, show the calculation of linkages (forward \& backward) and income multiplier. Use standard notations as used in the class notes. How can one classify the sectors between low and high linkages?
[6M]
Q19: Milly has decided to introduce a revolutionary video game. As the first firm in the market, it will have a monopoly position for at least some time. In deciding what type of technology is in production, it has the choice of two technologies: Technology $A$ is publicly available and will result in total costs of: $C^{A}(q)=10+8 q$. Technology $B$ is a proprietary technology developed in Milly's research labs. It involves a higher fixed cost of production but lower marginal costs. The total cost associated with this technology is: $C^{B}(q)=60+2 q$. Milly must decide which technology to adopt. Market demand for the new product is $P=20-Q$, where $Q$ is the total industry output. Suppose Milly was confident it would maintain its monopoly position in the market for the entire product lifespan without the threat of entry. a) Which technology would you advise Milly to adopt? b) What would be Milly's profit given this choice?
[6M]
Q20: Bata produces two types of leather shoes - T1 and T2. The contribution to profit per pair of shoes is INR 4 for T1 and INR 3 for T2. The time requirement for one pair of shoes of T1 is 2 hours and one pair of T 2 is 3 hours. The time available is sufficient to produce 500 pairs of shoes T1, and the leather is sufficient for only 400 pairs of shoes (both T1 and T2). T1 shoes require a fancy buckle, and only 200 fancy buckles are available. Formulate the above as a linear programming problem. Solve this using a graphical method. [8M]

# Birla Institute of Technology \& Science, Pilani, Pilani Campus - Rajasthan <br> Comprehensive Exam (Open Book) <br> ECON F 213 [Mathematical \& Statstical Methods] 

Maximum Marks: 40
Session 2023-24 (I)

## Instructions:

- Read the questions thoroughly before answering. All questions are compulsory. Start each question on a new page. Calculation(s) to arrive at the result(s) and its Interpretation are necessary to get marks.
- The calculator is allowed, but the exchange of the calculator is not.
- Ensure you correctly mention your Name, ID, Course, and other details on your answer sheet.
- During calculation, please consider two points after decimal without round-off.


## Part C

## Long Answer Type Question

Q1: As a chocolate maker, you must decide at what price to sell chocolate with rose flavor. You and your competitor across the street are selling the same chocolate, so you must consider the price at which others are selling to ensure you attract as many customers as possible. In addition to this, you also have the following information:

- If you and your competitor sell ₹ 20 , you will sell $60 \%$ of the chocolates out of the total sold in the market.
- If you sell at ₹20, but your competitor sells the shoes at ₹ 30 , you will sell $70 \%$ of the chocolates out of the total sold in the market.
- If you sell at ₹ 30 , but your competitor sells the shoes at ₹ 20 , you will sell $30 \%$ of the chocolates out of the total sold in the market.
- If you and your competitor sell ₹ 30 , you will sell $60 \%$ of the chocolates out of the total sold in the market.
a) Represent the above information as a game and answer the question: If your and your competitor's primary goal is to sell as many chocolates as possible, what is the best price to sell the chocolate at?
b) After doing research, you find that 80 people will be buying these chocolates. If you and your competitor's primary goal is to maximize the amount of money made from selling the chocolates, what is the best price to sell the chocolates at?
[5M]
Q2: A production function is given by $Q=4 L^{\frac{2}{3}} K^{\frac{1}{3}}$. Where the notations have their usual meanings. Answer the following:
a) Calculate the Degree of homogeneity and verify Euler's Theorem
b) Find the behavior of the marginal product of each factor
c) Examine the nature of returns to scale
d) Find the MRTS function
e) Find the share of capital and labor in total output
[15M]
Q3: A firm is interested in finding if there is any difference in the average salary received by workers of two different divisions, accordingly, samples of 12 workers in the first division and 10 workers in the second division were selected at random. The results are given below:

|  | Division 1 | Division 2 |
| :--- | :--- | :--- |
| Average Monthly Salary | 12500 | 11200 |
| Standard Deviation | 320 | 480 |

Test the hypothesis of whether there is a significant difference in the average salary of workers in the two divisions.
[10M]

# Birla Institute of Technology \& Science, Pilani, Pilani Campus - Rajasthan <br> Comprehensive Exam (Open Book) 

ECON F 213 [Mathematical \& Statstical Methods]
Q4: In an economy of three sectors, $\mathrm{P}, \mathrm{Q}$, and R, the data given below are available (All figures are in INR Million):

| Products | Users |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: |
|  | $\mathbf{P}$ | $\mathbf{Q}$ | $\mathbf{R}$ | FD | Output |
| $\mathbf{P}$ | 80 | 100 | 100 | 40 | 320 |
| $\mathbf{Q}$ | 80 | 200 | 60 | 60 | 400 |
| $\mathbf{R}$ | 80 | 100 | 100 | 20 | 300 |

Determine the output if the demand changes to 60 for $\mathrm{P}, 40$ for Q , and 60 for R .
[10M]
*******All the Best*****

