BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
FIRST SEMESTER 2022-2023
Mid – Semester Test

Course No.	: **ECON F242**	Max. Marks	**: 30**
Course Title	: **Microeconomics**	Weightage	**: 30%**
Date	: **02/11/2022**	Duration	**: 90 mints**

Q1. Suppose Johny has an income of \$12 to purchase and consume only two goods: Sodas and Sandwiches. The price of a bottle of soda is \$1, and the price of a single unit of sandwich is \$2. Assume Johnny's utility function is U (X, Y) = XY (X is the consumption amount of sodas and Y is the consumption amount of sandwiches). **[4M]**

a) Suppose the price of a bottle of soda increases to \$4, find his utility maximization consumption bundles before and after the change in price.
b) Calculate and graphically represent the income and substitution effect. (Note: Indicate Sodas along X axis)
c) At the new price level, at least how much income should Jack get (or give away) to achieve the original utility level?

Q2. State and prove the Hick's second law of demand. **[3M]**

Q3. Briefly explain the following: **[10M]**

a) Composite Commodity theorem
b) Graphically represent Income and Substitution effects for an Inferior good and Giffen good
c) Properties of Cost functions
d) Difference between Markup on Cost and Markup on Price by substantiating your answer citing an example
e) Producer surplus in short run

Q4.. In a perfectly competitive industry the market demand that can be represented as Q = -200P + 8000. All the firms in the market are having identical cost conditions and the total cost function for a representative firm is given as: $(q^3 / 300) + 0.2\, q^2 + 4q + 10$. Based on this information, now answer the following questions: **[5M]**

a) Derive the representative firm's short run supply function
b) Calculate the equilibrium price and quantity in this market if there are 100 firms in the industry in the short run.
c) Calculate the representative firm's short run profits / losses.

Q5. Suppose a firm uses two inputs in the production process i.e labor (L) and capital (K). The production function of the firm is given as: $Q (K, L) = L^{0.6}K^{0.4}$ **[8M]**

a) Mention the unconditional demand functions.
b) Derive and clearly mention the conditional demand functions.
c) Assume that initially total outlay of the firm is \$300, price of L = \$15 and price of K = \$20. Then subsequently the price of L changes to \$30. Calculate and mention the various types of effect for the given situation.
d) Graph the above situation as given in (c) clearly mentioning the various effects.
e) Graph the conditional and unconditional demand curves along with the respective values along with the original graph as given in (d).

*******END*******

Q1. Suppose Johny has an income of \$12 to purchase and consume only two goods: Sodas and Sandwiches. The price of a bottle of soda is \$1, and the price of a single unit of sandwich is \$2. Assume Johnny's utility function is U (X, Y) = XY (X is the consumption amount of sodas and Y is the consumption amount of sandwiches). **[4M]**

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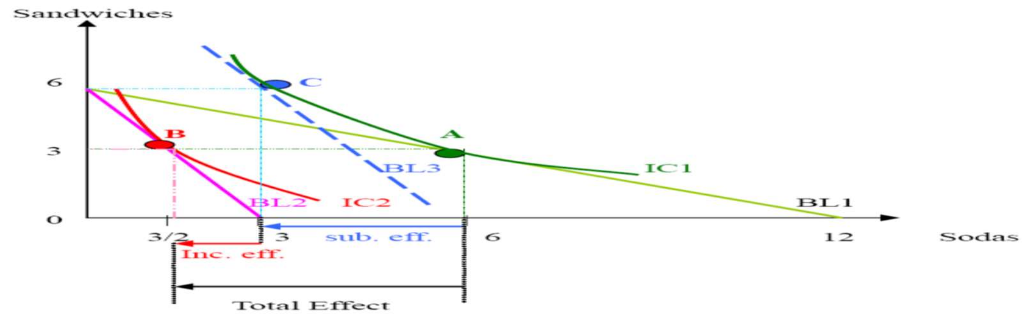


a) IE = 1.5 and SE = 3

b) Karthik's current income is only \$12, so he needs (24-12 = 12) dollars to achieve the original utility

Q2. State and prove the Hick's second law of demand. **[3M]**

According to Hicks' second law of demand, "most" goods must be substitutes

To prove this, we can start with the compensated demand function $x^c(p_1,\ldots p_n,V)$

Applying Euler's theorem yields

$$p_1 \cdot \frac{\partial x_i^c}{\partial p_1} + p_2 \cdot \frac{\partial x_i^c}{\partial p_2} + \ldots + p_n \frac{\partial x_i^c}{\partial p_n} = 0$$

In elasticity terms, we get $e_{i1}^c + e_{i2}^c + \ldots + e_{in}^c = 0$

- Since the negativity of the substitution effect implies that $e_{ii}^c \leq 0$, it must be the case that

$$\sum_{j \neq i} e_{ij}^c \geq 0$$

- The sum of all the compensated cross – price elasticities for a particular good must be positive (or zero).
- Thus most goods are substitutes.

Q3. Briefly explain the following: **[10M]**

a) Composite Commodity theorem:

Suppose that consumers choose among n goods. The demand for x_1 will depend on the prices of the other n-1 commodities. If all of these prices move together, it make sense to lump them into a single composite commodity (y)

Let $p_2^0\ldots p_n^0$ represent the initial prices of these other commodities assume they all vary together

Define the composite commodity y, such that $y = p_2^0x_2 + p_3^0x_3 + \ldots + p_n^0x_n$

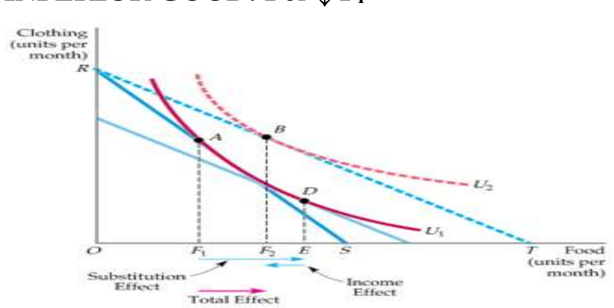
The individual's budget constraint is $I = p_1x_1 + p_2^0x_2 + \ldots + p_n^0x_n = p_1x_1 + y$

If we assume that all of the prices $p_2^0\ldots p_n^0$ change by the same factor ($t > 0$) then the budget constraint becomes

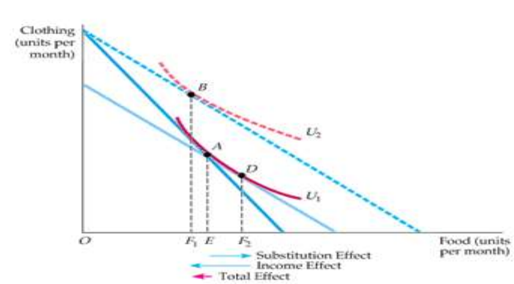
$\boldsymbol{I} = p_1x_1 + tp_2^0x_2 + \ldots + tp_n^0x_n = \boldsymbol{p_1x_1 + ty}$ where changes in p_1 or t induce substitution effects

b) Graphically represent Income and Substitution effects for an Inferior good and Giffen good

INFERIOR GOOD: For ↓ P_F



The Giffen Good

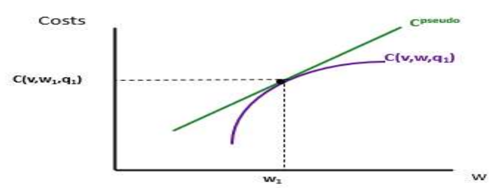


c) Properties of Cost functions

1) Homogeneity: Cost functions are all homogeneous of degree one in the input prices. A doubling of all input prices will precisely double the cost of producing any given output level. It will not change the levels of inputs purchased

2) Non decreasing in *q*, *v*, and *w*: cost functions are derived from a cost-minimization process. Any decline in costs from an increase in one of the function's arguments would lead to a contradiction

3) Concave in input prices: At w_1, the firm's costs are $C(v,w_1,q_1)$



If the firm continues to buy the same input mix as w changes, its cost function would be C^{pseudo}

Since the firm's input mix will likely change, actual costs will be less than C^{pseudo} such as $C(v,w,q_1)$

d) Difference between Markup on Cost and Markup on Price by substantiating your answer citing an example

A pricing technique whereby a certain percentage of cost of goods sold or of price is added to the cost of goods sold in order to derive the market price.

e) Producer surplus in short run

Producer surplus = $\Pi(P_1,\ldots) - \Pi(P_0,\ldots) = \Pi(P_1,\ldots) - (-vk_1) = \Pi(P_1,\ldots) + vk_1$

Producer surplus is equal to current profits plus short-run fixed costs

Q4.. In a perfectly competitive industry the market demand that can be represented as Q = -200P + 8000. All the firms in the market are having identical cost conditions and the total cost function for a representative firm is given as: $(q^3 / 300) + 0.2\,q^2 + 4q + 10$. Based on this information, now answer the following questions: **[5M]**

d) Derive the representative firm's short run supply function

e) Calculate the equilibrium price and quantity in this market if there are 100 firms in the industry in the short run.

f) Calculate the representative firm's short run profits / losses.

Q5. $C(q) = \dfrac{1}{300}q^3 + .2q^2 + 4q + 10 \qquad MC(q) = .01q^2 + .4q + 4$

a) For short run supply function set : $P = MC$ so that we get q = f(P)

$P = 0.01q^2 + .4q + 4 \qquad 100P = q^2 + 40q + 400 \quad$ or $(q + 20)^2 = 100P$,

The firm supply function is $q + 20 = 10\sqrt{P} \qquad q = 10\sqrt{P} - 20$ for **$P \geq 4$** **[4 marks]**

b) Industry supply: $Q = 100q = 1000\sqrt{P} - 2000$ while Demand: $Q = -200P + 8000$

Setting demand = supply $\quad -200P + 8000 = 1000\sqrt{P} - 2000$ or $1000\sqrt{P} + 200P = 10{,}000$

$5\sqrt{P} + P = 50, P = 25 \quad Q = 3000$ **[2 + 2 marks]**

c) For each firm ***q* = 30**, C = 400, AC = 13.3, **π = 351.** **[2 + 2 marks]**

Q5. Suppose a firm uses two inputs in the production process i.e labor (L) and capital (K). The production function of the firm is given as: $Q(K, L) = L^{0.6}K^{0.4}$ **[8M]**

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Q3. a) Unconditional demand functions are **L = 0.6C/ PL and K = 0.4C/ PK.** **[2M]**

b) For deriving conditional demand functions, first derive the

Indirect output function = **Q (PL, PK, TC) = $(0.6C/PL)^{0.6}$ x $(0.4C/PK)^{0.4}$**

Thus TC **or** $C = \frac{Q\,PL^{0.6}PK^{0.4}}{0.6^{0.6}0.4^{0.4}}$ $\quad$ **$L^C = \frac{0.6}{p_L}\left(\frac{Q\,PL^{0.6}PK^{0.4}}{0.6^{0.6}0.4^{0.4}}\right) = (1.5)^{0.4}(P_K/P_L)^{0.4}\,Q$**

And **similarly $K^c = \frac{0.4}{p_K}\left(\frac{Q\,PL^{0.6}PK^{0.4}}{0.6^{0.6}0.4^{0.4}}\right) = (2/3)^{0.6}(P_L/P_K)^{0.6}\,Q$** [1 + 1M]

c) Initially the firm uses (0.6 x 300) / 15 = 12 units of L and (0.4 x 300) / 20 = 6 units of K (Initial optimal allocation L = 12 and K = 6)

The firm's total output Q = $12^{0.6}$ x $6^{0.4}$ = 9.094

After the price of L increase the firm uses (0.6 x 300) / 30 = 6 units of L and 6 units of K. (Final optimal allocation L = 6 and K = 6)

The firm's use of L decrease by 6 units (from 12 to 6)

For calculating Substitution effect, keep output = 9.094 and equate MRTS = new price ratio, it gives,

3K / 2L = 30/20

We get K = L = 9.0943 (Decomposition basket)

Substitution effect = 9.0943 – 12 = - **2.906.** **[1M]**

Output effect = 6 - 9.0943 = - **3.0943** **[1M]**

d)and e) **[1 + 1M]**