Birla Institute of Technology and Science, Pilani.
Mid-Semester Examination: EEE F434 : Digital Signal Procssing
Marks: 60 AY: 2016-17, Semester: I Date: 8-October-2016, Saturday
Time: 90 minutes Pages: 02

Q1 Let $x[n] = [2, -1, 0, c]$ and $x_1[n] = [2, 2, -1, 0]$. $x[3] = c$ is an unknown constant. **[10]**

$X_1[k] = X[k]e^{j6\pi k/4}$ where $X[k]$and $X_1[k]$ are four-point DFT of $x[n]$ and $x_1[n]$ respectively.

a) Find the value of c?

b) Calculate and plot the four-point DFT $X[k]$

c) Calculate and plot the four-point DFT $X_1[k]$

d) Calculate $y[n] = x[n]$ © $x_1[n]$

e) Calculate $y[n]$ of part (d) by multiplying the DFTs of $x[n]$ and $x_1[n]$ and performing and inverse DFT.

Q2 Determine order (N), cut-off frequency (Ω_c) and $H(s)$ for a Butterworth filter for which $\delta_p = \frac{1}{\sqrt{2}}, \delta_s = 0.1, \Omega_p = 2\ rad/sec$ and $\Omega_s = 4\ rad/sec$. **[12]**

Determine the order (N) and $H(s)$ for a Chebyshev filter which satisfies the above given constraint.

Q3 The signal $y[n]$ is the output of an LTI system with impulse response $h[n]$ for an given input $x[n]$. Assume that $y[n]$ is stable and has z-transform $Y(z)$with the pole-zero plot shown in figure 3.1. The signal $x[n]$ is stable and has pole-zero plot shown in figure 3.2. **[10]**

a) What is ROC of $Y(z)$? Draw with shaded region.

b) Is $y[n]$ left sided, right sided or two sided?

c) What is ROC of $X(z)$? Draw with shaded region.

d) Is $x[n]$ causal sequence? Justify.

e) What is $x[0]$?

f) Draw the pole-zero plot of $H(z)$ and specify its ROC.

g) Is $h[n]$ causal or anticausal sequence? Justify.

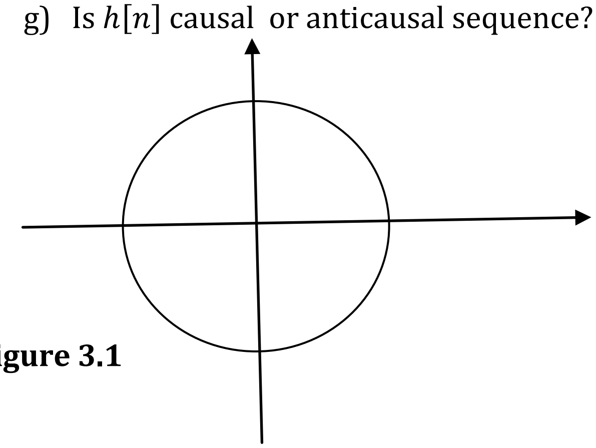

Figure 3.1

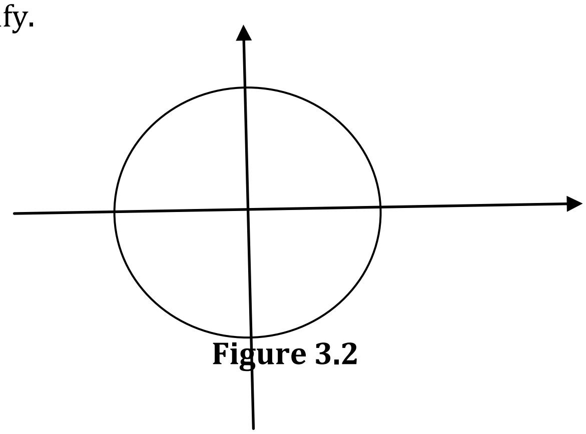

Figure 3.2

Q4 Consider the two systems shown in figure 4.1. **[16]**

a) Sketch the magnitude spectrum of $x[n]$, $y[n]$, $s_a(t)$ and $s[n]$ if $x_a(t)$ has a Fourier transform shown in figure 4.2. Assume $F_s = 2B$. Make an concluding remark.

b) Sketch the magnitude spectrum of $x[n], y[n], s_a(t)$ and $s[n]$ if $x_a(t)$ has a Fourier transform shown in figure 4.2 for following conditions:

- $F_{max} = 20\ Hz$ and $F_s = 50\ Hz$.
- $F_{max} = 20\ Hz$ and $F_s = 30\ Hz$

Note: $y[n] = x^2[n]$ *and* $s_a(t) = x_a^2(t)$

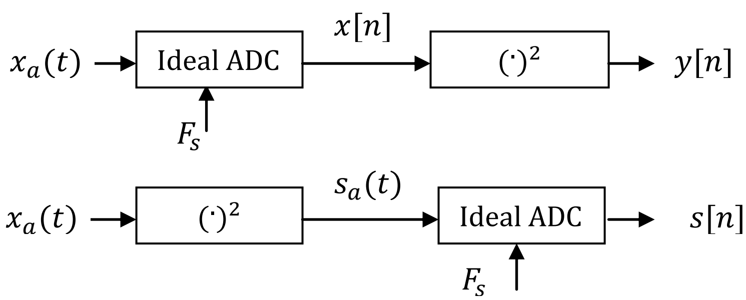

Figure 4.1

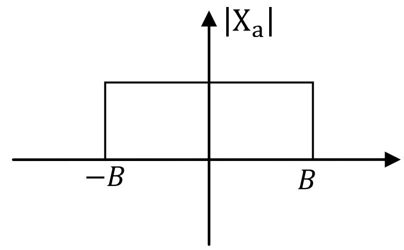

Figure 4.2

Q5 Let S_1 be a causal and stable LTI system with impulse response $h_1[n]$ and frequency response $H_1(e^{j\omega})$. The input $x[n]$ and output $y[n]$ for S_1 are related by the deference equation $y[n] - y[n-1] + \frac{1}{4}y[n-2] = x[n]$ **[12]**

a) If an LTI system S_2 has a frequency response given by $H_2(e^{j\omega}) = H_1(-e^{j\omega})$, would you characterize S_2 as being a low-pass filter, high-pass filter or a band-pass filter? Justify.

b) Let S_3 be a causal LTI system whose frequency response $H_3(e^{j\omega})$ has the property that $H_3(e^{j\omega})H_1(e^{j\omega}) = 1$

Is S_3 a minimum-phase filter? Could S_3 be classified as one of the four types of FIR filters with generalized linear-phase? Justify.

c) Let S_4 be a stable and noncausal LTI system whose frequency response is $H_4(e^{j\omega})$ and whose input $x[n]$ and output $y[n]$ are related by the deference equation

$$y[n] + \alpha_1 y[n-1] + \alpha_2 y[n-2] = \beta x[n]$$

where α_1, α_2 and β are all real and nonzero constants. Find α_1, α_2 and β such that $|H_4(e^{j\omega})| = |H_1(e^{j\omega})|$