Birla Institute of Technology and Science, Pilani.
Mid-Semester Examination: EEE F243/ INSTR F243: Signals and Systems
Marks: 60 AY: 2016-17, Semester: II Date: 09-March-2017, Thursday
Time: 90 minutes OPEN BOOK Pages: 02

Note: Neat and legible figures must be drawn wherever mentioned with all credentials.

Q 1 **a)** If $[n] = \{1, \underset{\uparrow}{1}, 1, 1, 1, \frac{1}{2}\}$. $\uparrow$ indicates the value at n=0. **15**

sketch $x[n]$, $x[n-2]$, $x[4-2n]$, $x\left[\frac{n}{2}\right]\left\{\frac{1}{2}\delta[n+2] - \delta[n-8]\right\}$.

b) Consider a system with input $x(t)$ and with output $y(t)$ given by

$$y(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT)$$

Is this system linear? Is this system time-invariant?

For each part, if your answer is yes, show why this is so. If your answer is no, produce a counter example.

c) Suppose that the input to this system is $x(t) = \cos 2\pi t$. Sketch and label carefully the input $x(t)$, output $y(t)$ for each of the following values of T: $T = \frac{1}{2}, \frac{1}{8}$.

[5+5+5=15M]

Q 2 a) $x[n]$ is a periodic signal with period N=8 and Fourier series coefficient a_k. Sketch one period of $x[n]$ from the information below: **15**

1. $a_k = -a_{k-4}$
2. $x[2n-1] = (-1)^n$.

b) Sketch the magnitude and phase of the frequency response of a linear, time-invariant system with the following unit impulse response:

$$h[n] = \delta[n] - \delta[n-3].$$

c) Sketch (one cycle of) the magnitude and phase spectrum of a periodic signal shown in figure 1 below:

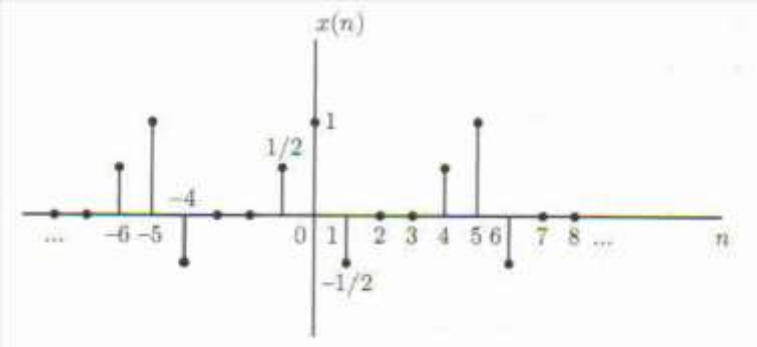

Figure 1: Periodic signal **[5+5+5=15M]**

Q 3 a) Consider the causal LTI system characterized by the difference equation below. Write an expression for the impulse response ($h_1[n]$) of this system. **15**

$$System\ 1:\ \ y[n] = -\frac{2}{3}y[n-1] + x[n] + \frac{8}{27}x[n-3]$$

b) Is the system causal? Is the system stable? Does the system have memory? Justify your answer using the impulse response.

c) Determine and write a closed-form expression for the output $y[n]$ of System 1 for the input $x[n] = 3\left(\frac{1}{3}\right)^n u[n]$.

d) Consider a second LTI system described by the following difference equation. Determine the impulse response ($h_2[n]$) for System 2. Write your answer in sequence form, using an arrow to denote the $n = 0$ value.

$$System\ 2:\quad y[n] = -y[n-1] + x[n] + x[n-5]$$

e) Determine the output $y[n]$ when input $x[n] = (n+1)\{u[n] - u[n-4]\}$ is applied to cascade combination of System 1 and System 2 described in (a) and (d) respectively. Write your answer in sequence form, using an arrow to denote the $n = 0$ value.

f) Sketch output $y[n]$.

[2+3+3+2+4+1=15M]

Q 4 Consider an LTI system with impulse response **15**

$$h(t) = \left\{\frac{\sin(5t)\sin(15t)}{\pi t^2}\right\} 2\cos(25t)$$

a) Determine and plot the frequency response $H(j\omega)$.

b) Determine the output $y(t)$ of the given LTI system for the input $x(t)$. The final answer ($\boldsymbol{y(t)}$) should be represented as sum of sinusoids.

$$x(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - n\frac{2\pi}{5}\right)$$

c) Sketch $X(j\omega)$ and $Y(j\omega)$.

[5+5+5=15M]