COMPREHENSIVE EXAMINATION: PART-A

Instructor: B. Sainath, 2210A, Electrical and Electronics Eng. Dept., BITS Pilani.

Course No./Title : EEE F311/Communication Systems   DATE: 9th Dec. 2016 (FN)

Max. marks: 30   Max. Time: 60 Mins.

Note: Write the right option and corresponding answer in your answer sheet. For each right answer, you get 2 marks. If your answer is incorrect, you lose 0.5.

Q. 1. Suppose that four signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and, $m_4(t)$ are to be multiplexed and transmitted. $m_1(t)$ and $m_2(t)$ have bandwidth of 4 KHz, and, the remaining two signals have bandwidth of 8 KHz. Furthermore, the system uses Nyquist sampling rate. If each sample is encoded to 8 bits, the minimum transmission bit rate of the system is

A. 16 Kbps   B. 192 Kbps   C. 1024 Kbps   D. 384 Kbps

Ans. D

Q. 2. Statement $\mathcal{S}$: With TDM, simultaneous transmission of several baseband signals on a time-sharing basis, is possible.

Reason $\mathcal{R}$: If the Nyquist sampling theorem is strictly followed, any continuous-time signal can be reconstructed from its samples.

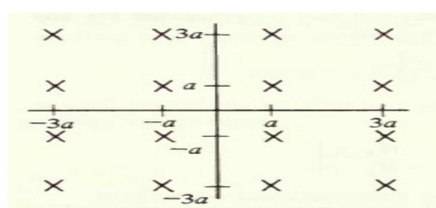
A. Both $\mathcal{S}$ and $\mathcal{R}$ are TRUE and $\mathcal{R}$ is the correct explanation for $\mathcal{S}$.

B. Both $\mathcal{S}$ and $\mathcal{R}$ are TRUE and $\mathcal{R}$ is NOT the correct explanation for $\mathcal{S}$.

C. $\mathcal{S}$ is TRUE but $\mathcal{R}$ is FALSE.   D. $\mathcal{S}$ is FALSE but $\mathcal{R}$ is TRUE.

Ans. B

Q. 3.. The signal-space diagram of a modem is shown in the figure. If the baud rate is 4 Msps, what is the bit rate ?



A. 4 Mbps   B. 40 Mbps

C. 16 Mbps   D. 64 Mbps

Ans. C

Q. 4. Which of the following statements about the matched filter are correct.

i). Its impulse response depends on its input signal shape.

ii). It maximizes SNR at the detection instant. iii). It produces intersymbol interference (ISI)

iv). It may produce phase error if proper synchronization is not maintained.

Select the right answer from the options given below.

A. i), iii) only   B. i) and ii) only   C. ii), iii), and, iv)   D. i), ii), and, iv)

Ans. B

Q. 5. Suppose that $Y(t)$ denote a widesense stationary (WSS) random process. The autocorrelation function $R_Y(\tau)$ has the property that $R_Y(0)$ is equal to

A. The square of the mean value of the process. B. The smallest value of $R_Y(\tau)$.

C. The mean squared value of the process. D. $\frac{R_Y(\tau)+R_Y(-\tau)}{2}$.

Ans. C

Q. 6. An audio signal is band-limited to 4 KHz. It is sampled at the Nyquist rate. The amplitude spectrum of the instantaneously sampled signal will be

A. between -8 KHz to 8 KHz B. between -4 KHz to 4 KHz

C. At every 4n KHz (n = 0, ± 1, ± 2, $\ldots$,..) D. At every 8n KHz (n = 0, ± 1, ± 2, $\ldots$,..)

Ans. D

Q. 7. Consider a binary symmetric channel (BSC) with crossover probability $p = 0.5$. The channel capacity and the entropy function, respectively, would be,

A. 1 and 0.5 B. 1 and 1 C. 0.5 and 1 D. 0 and 1

Ans. D

Q. 8. Match List-I (theorem) and List-II (specified quantity).

List-I: (a). Source coding theorem (b). Wiener-Khinchin theorem (c). Shannon-Hartley theory

List-II: (i). PSD of random process (ii). Channel capacity (iii). Optimum code length

Ans. $(a) \rightarrow (iii)$, $(b) \rightarrow (i)$, and $(c) \rightarrow (ii)$

Q. 9. Which of the following codes are error control codes ?

i). Hamming code ii). Huffman code iii). Convolutional code iv). Shannon-Fano code

A. i) and ii) only B. ii) and iii) only C. iii) and iv) only D. i) and iii) only

Ans. D

Q. 10. A coherent BPSK communication system operates at a bit rate of 10 Mbps. The received SNR is 10 dB. The values of bit energy-to-noise spectral density ratio and approximate symbol error probability (SEP), respectively, are

A. 10 dB, 3.9×10^{-5} B. 1 dB, 3.9×10^{-6} C. 10 dB, 3.9×10^{-6} D. 20 dB, 3.9×10^{-5}

Ans. C. 10 dB, $Q(4.47)$

Q. 11. An analog signal is band-limited to 4 KHz. It is sampled at the Nyquist rate. The samples are quantized into 4 levels which have probabilities $\frac{3}{8}$, $\frac{3}{8}$, $\frac{1}{8}$,, and, $\frac{1}{8}$. The approximate entropy and information rate of the source, respectively, are

A. 1.8 bits/sample, 14400 bps B. 1.8 bits/sample, 7200 bps

C. 0.9 bits/sample, 14400 bps D. 1.8 bits/sample, 28800 bps

Ans. A

Q. 12. A discrete memoryless source produces symbols with probabilities $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{8}$. For this source, a practical coding scheme has an average code word length of 2.0 bits per symbol. The code efficiency and the redundancy of the code, respectively, are

A. 1, 0 B. $\frac{7}{8}$, $\frac{1}{8}$ C. 0.5, 0.5 D. $\frac{1}{4}$, $\frac{3}{4}$

Ans. B

Q. 13. Consider a square pulse $p(t), 0 \leq t \leq 1$ sec. having amplitude $\sqrt{2}$ volt. The pulse is applied to a matched filter. Assuming that the noise power is normalized to unity, the output SNR is approximately

A. 3 dB B. -3 dB C. 0 dB D. 2 dB

Ans. A

Q. 14. Suppose that Y is the sum of twelve uniformly distributed random variables $\mathcal{U}_j[-\frac{1}{2}, \frac{1}{2}], j = 1, 2, \ldots, 12$. The mean and mean square value of Y, respectively, are

A. 0, 12 B. 1, 12 C. 0, 1 D. 12, 12

Ans. C

Q. 15. Let the baseband signal $m(t) = \sin(200\pi t) + 2\sin(400\pi t)$. The message signal is modulated to produce a signal $s(t) = m(t)\sin(400\pi t)$. This is passed through an ideal lowpass filter having angular cutoff frequency of 400π Hz and passband gain of 2. Then the output of the filter is

A. $0.5\cos(100\pi t)$ B. zero C. $2\cos(100\pi t)$ D. $\cos(200\pi t)$

Ans. D

□ END OF PART-A □

COMPREHENSIVE EXAMINATION: PART-B

Instructor: B. Sainath, 2210A, Electrical and Electronics Eng. Dept., BITS Pilani.
Course No./Title : EEE F311/Communication Systems DATE: 9th Dec. 2016 (FN)
Max. marks: 50 Max. Time: 120 Mins.

Important instructions:

- This part of the test is open book. Mention this on the top of your answer sheet.
- Write your ID number, name, tutorial section number, and instructor's name on your answer sheet.
- You can answer questions in any order. But, provide answers to all subparts within a question at one place. Give all your answers with proper units and do not skip intermediate steps.
- In all sketches, include x-label, y-label, figure caption. Incomplete figures will not get full credit.

Q. 1. [$M-$ary Orthogonal Signaling]

Suppose that a communication system uses M orthogonal signals $s_1, s_2, \ldots, s_M$ to communicate over AWGN channel with covariance matrix $\mathcal{K} = \frac{N_0}{2}\mathcal{I}_M$, where $\mathcal{I}$ denote $M \times M$ identity matrix. Let $\{\psi_j, j = 1, 2, \ldots, M\}$ denote the set of orthonormal basis functions. Furthermore, let each signal has energy $\mathcal{E}$.

Answer the following questions.

a). Give mathematical model for the M orthogonal signal set using the orthonormal basis set and prove that they are orthogonal. [2 marks]

Ans. The mathematical model for the M orthogonal signal set is given by

$$s_j = \sqrt{\mathcal{E}}\psi_j, j = 0, 1, \ldots, M-1.$$

To prove orthogonality, consider the inner product of any pair of signals s_j and s_k, $j \neq k$. Since ψ_j and ψ_k are orthonormal, we have

$$\begin{aligned} < s_j, s_k > &=< \sqrt{\mathcal{E}}\psi_j, \sqrt{\mathcal{E}}\psi_k >, \\ &= \sqrt{\mathcal{E}} < \psi_j, \psi_k >, \\ &= 0. \end{aligned}$$

b). Sketch the signal space diagram when M = 3. Indicate the three signal points and orthonormal basis. [2 marks]

Ans. The three dimensional constellation is shown in the Figure 1.

c). What is the Euclidean distance between any two vectors ? Was it independent of which pair of signals we choose ? Why ? [2 marks]

Ans. Euclidean distance = $\sqrt{2E}$. Yes. Due to orthogonality, the distance is independent of the pair of signals we choose.

Let $\mathcal{P}\left(E|m_k\right)$ denote the conditional probability of error, given that the k^{th} message m_k is transmitted. The union bound is given by the following inequality.

$$\mathcal{P}\left(E|m_k\right) \leq \sum_{j=1}^{M} Q\left(\frac{d_{kj}}{2\sigma}\right), j \neq k,$$

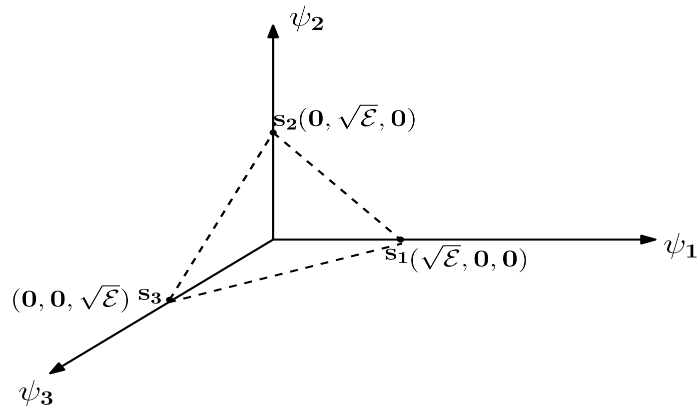


Fig. 1: $M-$ary FSK constellation (M = 3).

where d_{kj} is the Euclidean distance between vectors s_k and s_j, σ^2 is the AWGN two-sided power spectral density (PSD).

d). i). Using the above union bound, derive upper bound for probability of error p_e in terms of M, $\mathcal{E}$, and the noise PSD. Note that all messages are equally likely. [4 Marks]

Ans. Given that the messages are equally likely. Hence, $\mathcal{P}(m_j) = \frac{1}{M}, j = 1, 2, \ldots, M$. The probability of error is given by $p_e = \mathcal{P}(m_1)\mathcal{P}(E|m_1) + \mathcal{P}(m_2)\mathcal{P}(E|m_2) + \ldots + \mathcal{P}(m_M)\mathcal{P}(E|m_M)$. Using the upper bound, we get

$$\begin{aligned} p_e &= \mathcal{P}(m_1)\mathcal{P}(E|m_1) + \mathcal{P}(m_2)\mathcal{P}(E|m_2) + \ldots + \mathcal{P}(m_M)\mathcal{P}(E|m_M), \\ &\leq (M-1)Q\left(\sqrt{\frac{\mathcal{E}}{N_0}}\right). \end{aligned}$$

d). ii). For $M = 2$, compare the probability of error and its upper bound. Are they different ? [2.5 Marks]

Ans. For $M = 2$, the probability of error is given by

$$\mathcal{P}(E) = Q\left(\sqrt{\frac{\mathcal{E}}{N_0}}\right).$$

Therefore, the probability of error and its upper bound are equal.

Q. 2. [Wideband FM Power Components Computation] [12.5 marks]

Suppose that a 50 MHz sinusoidal carrier delivers 20 dBwatt power to a load. The carrier is frequency modulated by a 1 KHz message signal causing a peak frequency deviation of 6 KHz. The resultant FM signal is now coupled to the load through an ideal bandpass filter with 50 MHz center frequency a variable bandwidth.

Compute the power (in watts) delivered to the load when the filter bandwidth is i) 1 KHz ii) 2.1 KHz iii) 12.5 KHz iv) 14.5 KHz v) 20.2 KHz. (*Hint:* Determine the frequency components that are present and compute power in each case.)

Tabulate all the powers associated with all frequency components (including dc), and, power delivered to the load in each case, as shown below.

n	$f_c \pm nf_m$	Power (watts)

Case	Power delivered (in watts)

Given: $J_0(6) = 0.1506, J_1(6) = -0.2767, J_2(6) = -0.2429, J_3(6) = 0.1148, J_4(6) = 0.3576, J_5(6) = 0.3621, J_6(6) = 0.2458, J_7(6) = 0.1296, J_8(6) = 0.0565, J_9(6) = 0.0212, J_{10}(6) = 0.0070$. Here $J_n(\beta)$ denotes Bessel the function of first kind.

Ans. Given: Carrier frequency $f_c = 50$ MHz. Message signal frequency $f_m = 1$ KHz. Peak frequency deviation $\Delta f = 6$ KHz. Carrier power = 100 watt. Modulation index $\beta = \frac{\Delta f}{f_m}$ = 6.

Refer to the following tables.

n	$f_c \pm nf_m$	Power (watts)
0	50 MHz	2.268
1	$50MHz \pm 1KHz$	2(7.656)
2	$50MHz \pm 2KHz$	2(5.900)
3	$50MHz \pm 3KHz$	2(1.318)
4	$50MHz \pm 4KHz$	2(12.788)
5	$50MHz \pm 5KHz$	2(13.112)
6	$50MHz \pm 6KHz$	2(6.042)
7	$50MHz \pm 7KHz$	2(1.679)
8	$50MHz \pm 8KHz$	2(0.319)
9	$50MHz \pm 9KHz$	2(0.0449)
10	$50MHz \pm 10KHz$	$2(4.9 \times 10^{-3})$

Case (bandwidth)	Power delivered (in watts)
1 KHz	2.27
2.1 KHz	17.58
12.5 KHz	95.90
14.5 KHz	99.26
20.2 KHz	99.99

Q. 3. [Matched Filter] ***Part−I:*** A binary baseband communication system transmits one bit for every T_b seconds with $T_b = 4$. The pulse shape used is shown in figure 2.

i). The pulse $s(t)$ is applied as input to the matched filter. Sketch the impulse response $h(t)$ of the matched filter. [2.5 Marks]

ii). Determine the sampled output of the matched filter at t = 4. [2.5 marks]

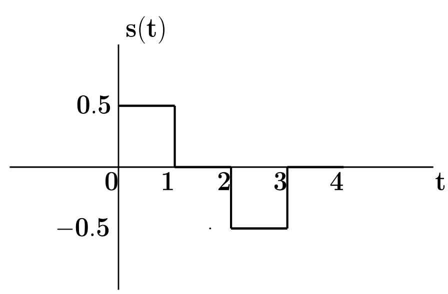


Fig. 2: Pulse shape in Q.3.

i). Ans. The impulse response of the matched filter is shown below.

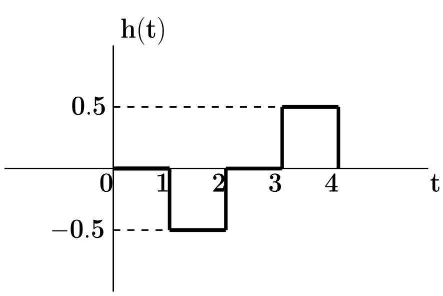


Fig. 3: Impulse response of the matched filter for the pulse shape in Q.3.

ii). Ans. Sampled output of the matched filter at t = 4 is 0.5.

Consider the data transmission over baseband channel using polar signaling in which bit '0' is mapped to -1 volt and bit '1' is mapped to $+1$ volt. The line encoded signal is transmitted over baseband channel. The received signal is corrupted by the AWGN.

iii). If the matched filter is used at the receiver, determine the SNR of the sampled output. [2.5 marks]

iv). Determine the probability of a bit error p_{be} of the matched filter receiver. Express p_{be} in terms of the complementary error function. [2.5 marks]

iii). Ans. The SNR of the samples output = $\frac{E_s}{N_0} = \frac{1}{2N_0}$.

iv). Ans. the probability of a bit error p_{be} of the matched filter receiver is given by $p_{\text{be}} = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{\frac{1}{N_0}}\right)$. In terms of the complementary error function, we have $p_{\text{be}} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{1}{2N_0}}\right)$.

Part–II: Consider the orthonormal basis functions $\psi_j, j = 1, 2, \ldots, M$, which are defined in the finite interval $0 \leq t \leq T$. Let $x(t)$ is a common input to a bank of filters. Furthermore, j^{th} filter's impulse response is denoted by $h_j(t)$. If we choose the filter whose response is matched to the basis function, derive expression for the output at t = T, denoted by $y_j(T)$. [2.5 marks]

Ans. When $x(t)$ is the filter input and $h_j(t)$ is its impulse response, we have

$$y_j(t) = \int_{-\infty}^{\infty} x(u)h_j(t-u)\,du,$$

where $y_jt)$ is the output of the filter. Now set $h_j(t) = \psi_j(T-t)$. Then the output is

$$y_j(t) = \int_{-\infty}^{\infty} x(u)\psi_j(T-t+u)\,du,$$

Finally, the output sampled at time $t = T$ is

$$y_j(T) = \int_{-\infty}^{\infty} x(u)\psi_j(u)\,du,$$

which is mathematically equivalent to correlation receiver's output.

Q. 4. [Source coding & Shannon-Fano Code] ***Part-I:*** a). When raw binary data generated by the source can be transmitted via a channel, why source coding is used in a digital communication system which improves complexity of transmission. ? What is source coding theorem and what it tells you ? (Note: Answer in TWO sentences.) [1 mark]

Ans. Source coding is used to achieve efficient representation of data generated by the discrete memoryless source in spite of the additional complexity introduced by the compression technique. Source coding theorem is a mathematical tool which conveys that the average length of the code word can be made no less than the entropy of the source.

b). A discrete memoryless source produces 8 distinct symbols in the set $\mathcal{S} = \{s_1, s_2, \ldots, s_8\}$ with corresponding probabilities $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{128}\}$. These symbols are encoded as 000, 001, 010, 011, 100, 101, 110, and, 111, respectively.

Answer the following:

i). What is the average amount of information per symbol ? [1 mark]

Ans. Entropy = 1.9844 bits/symbol.

ii). What is the probability of occurrence of 0 ? What is the probability of occurrence of 1 ? [2 marks]

Ans. The probability of occurrence of 0, $p_0 = 0.7995$. The probability of occurrence of 1, $p_1 = 0.2005$.

Part-II: Consider a DMS produces three distinct symbols $\mathcal{A} = \{\alpha_1, \alpha_2, \alpha_3\}$ with probabilities 0.5, 0.4, and, 0.1, respectively.

i). Obtain Shannon-Fano code. Compute its entropy, average code word length, efficiency, and, redundancy. [1.5 + 2 marks]

Ans. Code = $\{0, 10, 11\}$, Entropy $H(\mathcal{A}) = 1.361$ bits/symbol. Average code length $\overline{L} = 1.5$ bits/symbol. Code efficiency = 90.73% and redundancy 9.27%.

ii). Consider the second order extension of $\mathcal{A}$. Obtain probabilities of all symbols. Obtain Shannon-Fano code it. Determine its entropy, average code word length, efficiency, and redundancy. [3 + 2 marks]

Ans. Code = $\{11, 101, 100, 10, 001, 00011, 00010, 00001, 00000\}$, Entropy $H(\mathcal{A}) = 2.7219$ bits/symbol. Average code length $\overline{L} = 2.83$ bits/symbol. Code efficiency = 96.18% and redundancy 3.82%.

□ **END OF PART-B** □