Mid-Semester Test

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Course No./Title : EEE F311/Communication Systems   DATE: 6th Oct. 2016

Max. marks: 50   Test duration: 90 Mins.

Important instructions:

- Exam is fully closed book.
- Write correct tutorial section number as well as instructor's name on your answer book.
- You can answer questions in Part-III in any order. But, provide answers to all subparts within a question at one place.
- Each question is marked with the number of marks assigned to that question.
- Give all your answers with proper units.
- In all sketches, include x-label, y-label, figure caption. Incomplete figures will not get full credit.

Part-I: Multiple choice questions (8 marks)

Note: Each question carries ONE mark. Write the right answer in your answer book.

1. Let $m(t)$ denote the message signal and the carrier $c(t) = \cos(2\pi f_c t)$. The in-phase and quadrature components of double sideband suppressed carrier (DSB-SC), respectively, are

A. 0, $m(t)$ B. $m(t)$, 0   C. $m(t)$, $-m(t)$ D. $m(t)$, $-\hat{m}(t)$

Data for Q. (2), (3), (4), and (5):

Let U be a uniformly distributed random variable, defined by its probability density function (pdf)

$$p_U(u) = \begin{cases} 0.5, & -1 \leq u \leq 1, \\ 0, & otherwise \end{cases}.$$

Let the random variable $U_1 = U$ and the random variable $U_2 = U^2$.

Answer the following questions (2), (3), (4), and (5).

2. The mean value of the random variable U_1 is equal to

A. 1 B. 0.5 C. -0.5 D. 0

3. The mean value of the random variable U_2 is equal to

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. 0 4. The covariance of U_1 and U_2 is

A. 0 B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. undefined

5. The random variables U_1 and U_2 are

A. Statistically independent and uncorrelated. B. Statistically independent and correlated.

C. Uncorrelated and orthogonal. D. Uncorrelated but not orthogonal

6. The basic building blocks of a phase lock loop (PLL) are phase comparator,

A. low-pass filter (LPF) and voltage controlled oscillator (VCO).

B. High-pass filter (HPF) and VCO.

C. LPF and crystal oscillator.

D. HPF and crystal oscillator.

7. The modulation index of an AM wave is changed from zero to 1. The transmitted power is

A. increased by 50%. B. unchanged C. halved. D. quadrupled.

8. Match List-I (type of noise) and List-II (its property).

List-I: (a). Thermal noise (b). White noise (c). Narrow-band noise

List-II: (i). Power spectral density is independent of frequency (ii). Noise generated in resistor

(iii). Noise at output of frequency-selective filter

Part-II: True or False (2 marks)

Note: Each question carries $\frac{1}{2}$ point. Just indicate 'T/F' ('T' for True statement; 'F' for false statement.).

1) A Gaussian random process is completely specified by its expectation and autocorrelation.
2) If two random variables X and Y are uncorrelated, then they are statistically independent.
3) The power spectral density of wide-sense stationary random process is an even function.

4) Unlike an AM system, the transmitted power in an FM system is completely independent of the modulating signal.

Part III: Solve the following.

Q. 1. Consider a Gaussian distributed random variable (RV) W of zero mean and variance $\sigma^2 = 1$. The RV W is transformed by a piecewise-linear rectifier block characterized by the following input-output relation (refer to Figure 1):

$$Y = \begin{cases} W, & W \geq 0, \\ 0, & W < 0 \end{cases}.$$

Answer the following questions.

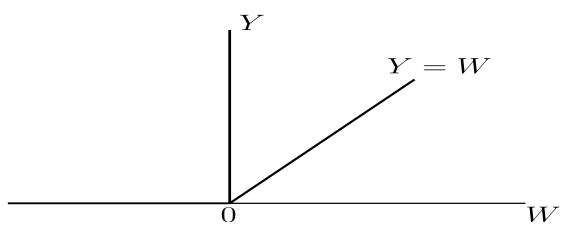


Fig. 1: Input-output characteristic of linear rectifier.

(a). For negative values of W, what is the output of the rectifier ? [1 mark]

(b). Let $p_Y(y)$ denote probability density function (pdf) of Y. For positive values of W, what is the pdf of Y? Write down the expression. [1 mark]

(c). What is the probability that $W < 0$? What is the probability that $Y = 0$? [1+1 marks]

(d). Write down the expression for the total pdf of Y. Sketch it. [1+2 marks]

(e). Sketch the cumulative distribution function (CDF) and complementary CDF (CCDF) of Y. [1+1 marks]

(f). Comment on nature of the output RV Y. [1 point]

Q. 2. Consider the filter shown in Figure 2. It consists of a delay line and a summing device.

Answer the following the questions.

a). Write down the expression for $Y(t)$. [1 mark]

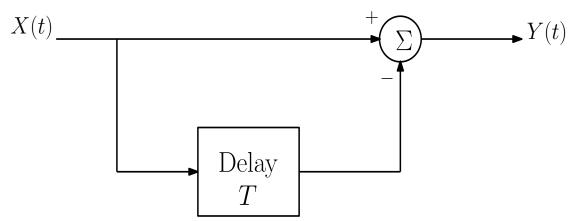


Fig. 2: Block diagram of comb filter.

b). Derive the transfer function $H(f)$ of the filter. Express the transfer function in terms of real part and imaginary part. [1+1 marks]

c). Derive the simplified expression for the squared magnitude of $H(f)$ and sketch it. [1+1 marks]

(d). Let $S_X(f)$ denote power spectral density (PSD) of $X(t)$. Express the output PSD $S_Y(f)$. Write down approximate expression for $S_Y(f)$ when $\pi f T$ very small. [1+1 marks]

(e). If the input $X(t)$ is white noise, find $S_Y(f)$, and approximate $S_Y(f)$ when $\pi f T$ very small. [1+1 marks]

(f). For low frequency inputs, the comb filter acts as integrator or differentiator ? Justify your answer. [1 mark]

Q. 3. Consider a modulating signal $m(t) = 10\sin(2\pi \times 10^4 t)$ volt. A sinusoidal carrier of frequency 2.5×10^7 Hz and peak amplitude $100\sqrt{2}$ volt is frequency modulated by $m(t)$. Furthermore, the maximum frequency deviation is 100 KHz. Recall that frequency modulated signal $s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau)$.

Answer the following questions.

Part (A) [5 marks]: (i). Compute frequency sensitivity. (ii). Compute the modulation index.

(iii). Write down expression for the FM signal. (iv). Compute transmitted signal power in dB.

(v.). Compute transmission bandwidth of the FM signal using Carson's rule.

Part (B) [2.5 marks]: Repeat Part (A) when modulating signal frequency is doubled.

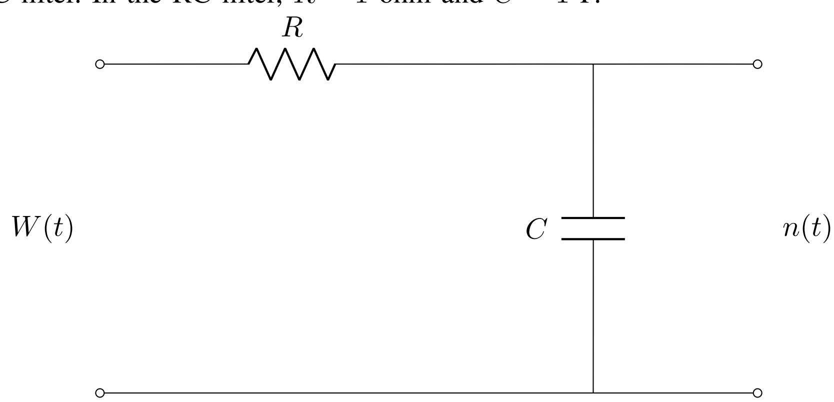
Part (C) [2.5 marks]: In this scenario, frequency sensitivity is fixed. But, peak amplitude of original modulating signal is halved and frequency is doubled. Furthermore, peak carrier amplitude is quadrupled.

(i). Compute peak frequency deviation. (ii). Compute the modulation index.

(iii). Write down expression for the FM signal. (iv). Compute transmitted signal power in dB.

(v.). Compute transmission bandwidth of the FM signal using Carson's rule.

Q. 4. Consider a white Gaussian noise $W(t)$ of zero mean and spectral density $\frac{N_0}{2}$ applied to a low-pass RC filter. In the RC filter, $R = 1$ ohm and $C = 1$ F.



(a). Write down the transfer function of the filter $H(f)$ [1 mark]

(b). What is the dc gain of the system. [1 mark]

(c). Derive spectral density of output random process and sketch it. [2+1 marks]

(d). Derive autocorrelation function of output random process and sketch it. [2+1 marks]

(e). What are the mean value and the mean square value of the output random process ? [1+1 marks]

□ END OF QUESTION PAPER □

Answers

Part I

1. B 2. D 3. B 4. A 5. C 6. A 7. A 8. $(a) \to (ii)$, $(b) \to (i)$, and, $(c) \to (iii)$

Part II

1. T 2. F 3. T

4. T (Wideband FM assumed).

F (In case of narrowband FM).

Part III

Q. 1. Ans. (a). For negative values of W, the output of the rectifier zero.

(b). For positive values of W, the pdf of Y is given by

$$p_y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right), y > 0.$$

(c). From the symmetry of Normal distribution, the probability that $W < 0$ is $\frac{1}{2}$ which corresponds to the probability that $Y = 0$. Therefore, we have $\mathcal{P}(W < 0) = \frac{1}{2}$ and $\mathcal{P}(Y = 0) = \frac{1}{2}$.

(d). Total probability density function of Y can be expressed as

$$p_Y(y) = \begin{cases} \frac{1}{2}\delta(y) + \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right), & y \geq 0, \\ 0, & y < 0 \end{cases}.$$

The pdf is plotted in Figure 3.

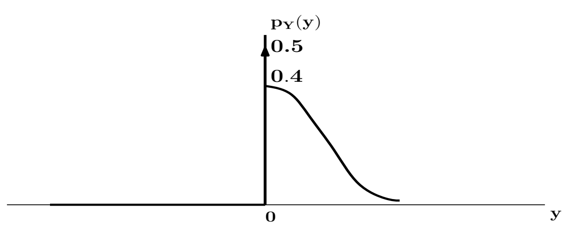


Fig. 3: Probability density function of the random variable Y at the output of a piecewise-linear rectifier with normal random variable as input.

(e). The CDF of the random variable Y are shown in Figure 4. The CCDF of the random variable Y

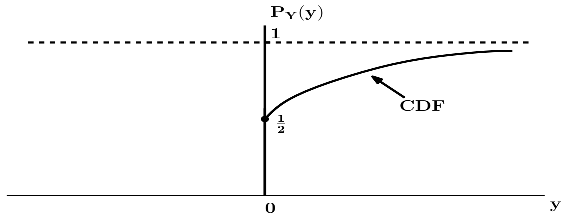


Fig. 4: Cumulative distribution function (CDF) and CCDF of the random variable Y.

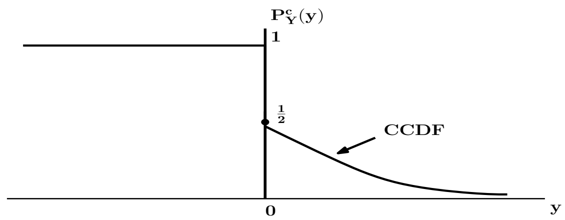


Fig. 5: CCDF of the random variable Y.

are shown in Figure 5.

(f). Notice that this probability density function of Y is mixed in which both continuous and discrete components are present.

Q. 2. ans. (a). The expression for $Y(t)$ is given by $Y(t) = X(t) - X(t-T)$.

(b). The transfer function $H(f)$ of the filter is given by $H(f) = (1 - \cos(2\pi fT)) + j\sin(2\pi fT)$.

(c). The expression for the squared magnitude of $H(f)$ is given by $|H(f)|^2 = 4\sin^2(\pi fT)$. Normalized $|H(f)|^2$ is shown in Figure 6 for $T = \frac{1}{\pi}$ second.

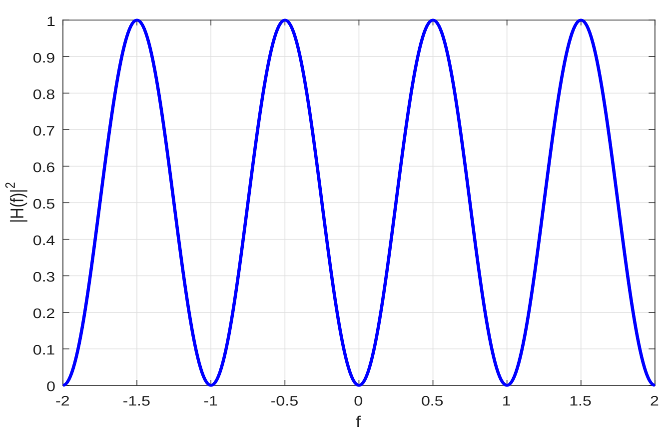


Fig. 6: Frequency response of comb filter.

(d). Exact output spectral density is $S_Y(f) = 4\sin^2(\pi f T)S_X(f)$. For fT small, we have $S_Y(f) = 4\pi^2 f^2 T^2 S_X(f)$.

(e). We have $S_Y(f) = \frac{N_0}{2}$. Therefore, exact output spectral density $S_Y(f) = 2N_0 \sin^2(\pi f T)$. For fT small, we have $\sin(\pi f T) \approx \pi f T$. Therefore, $S_Y(f) \approx 2N_0\pi^2 f^2 T^2$.

(f). Recall that differentiation in the time domain corresponds to multiplication by $j2\pi f$ in the frequency domain. We observe that the comb filter acts as a differentiator for low-frequency inputs.

Q. 3. ans. *Part (A):* (i). Frequency sensitivity $K_f = 10^4$ Hz/V.

(ii). The modulation index $\beta = 10$ which is wideband FM (WBFM).

(iii). The expression for the FM signal is given by $s(t) = 100\sqrt{2}\cos(50\pi \times 10^6 t - 10\cos(2\pi \times 10^4 t) + k)$ volt. Note that the constant 'k' will not effect FM signal parameters such as instantaneous frequency, peak frequency deviation, modulation index, bandwidth, and, transmitted power. Hence, the constant is omitted in the rest of the FM signal expressions.

(iv). Transmitted signal power $P_t = 40$ dB. Recall that, for WBFM, transmitted signal power is equal to carrier power.

(v.). The transmission bandwidth of the FM signal using Carson's rule: $B_T = 2(\beta + 1)f_m = 220$ KHz.

Part (B): Repeat Part (A) when modulating signal frequency is doubled.

(i). Unchanged. That is, frequency sensitivity $K_f = 10^4$ Hz/V.

(ii). Modulation index $\beta = 5$. Note that this is wideband FM signal.

(iii). FM signal $s(t) = 100\sqrt{2}\cos(50\pi \times 10^6 t - 5\cos(4\pi \times 10^4 t))$ volt.

(iv). Unchanged. That is, transmitted signal power $P_t = 40$ dB.

(v). Transmission bandwidth $B_T = 240$ KHz.

Part (C):

(i). Peak frequency deviation $\Delta f = 50$ KHz and modulation index $\beta = 2.5$. This is again wideband FM signal.

(ii). FM signal is given by $s(t) = 400\sqrt{2}\cos(50\pi \times 10^6 t - 2.5\cos(2\pi \times 10^4 t))$ volt. Transmitted signal power $P_t = 52$ dB.

(iii). The transmission bandwidth of the FM signal using Carson's rule: $B_T = 140$ KHz.

Q. 4. ans. (a). The transfer function $H(f)$ of the filter is given by $H(f) = \frac{1}{1+j2\pi f}$.

(b). The dc gain of the system = $H(f)$ at $f = 0$ which is equal to 1.

(c). Spectral density of output random process is given by

$$S_N(f) = \frac{N_0}{2(1 + 4\pi^2 f^2)}.$$

The spectral density is shown in Figure 7.

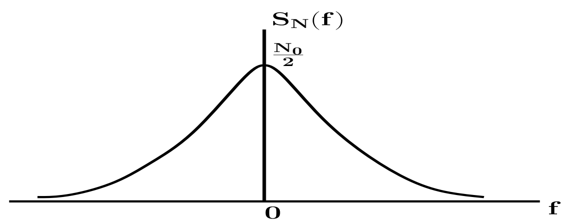


Fig. 7: Spectral density of RC low-pass filter output $n(t)$.

(d). Recall that the inverse Fourier transform of power spectral density is autocorrelation function. The autocorrelation function of filtered noise is given by

$$R_N(\tau) = \frac{N_0}{4}\exp\left(-|\tau|\right).$$

The spectral density is shown in Figure 8.

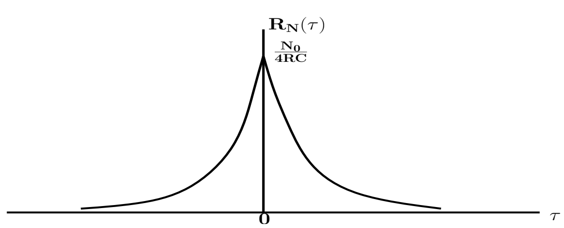


Fig. 8: Autocorrelation function of RC low-pass filter output $n(t)$.

(e). The mean value (MV) of output is zero. The mean square value (MSV) of the output is $\frac{N_0}{4}$.