COMPREHENSIVE EXAMINATION: PART-A

Instructor-in-Charge: B. Sainath, 2210A, Electrical and Electronics Eng. Dept., BITS Pilani.

Course No./Title : EEE F311/Communication Systems   DATE: 7th Dec. 2017 (FN)

Max. marks: 30   Max. Time: 60 Mins.

Note: Write the right option and corresponding answer on your answer sheet provided. For each right answer, you get **2** marks. If your answer is incorrect, you lose **0.5** mark. Over-written answers will not be evaluated and rechecked. Make sure that the question paper and answer sheet have the same SET label.

Q. 1. Consider a noisy resistor. The mean square value of thermal noise current in a frequency band 'B' is i). directly proportional to the conductance, ii). inversely proportional to the bandwidth, and, iii). directly proportional to the room temperature.

A. All are true   B. Only (i) is true   C. (ii) and (iii) are true   D. (i) and (iii) are true.

Q. 2. Consider a discrete memoryless source that emits K symbols. The symbols are transmitted through a noiseless discrete channel for which $p(x_i|y_j) = 1$ for $i = j$ and $p(x_i|y_j) = 0$ for $i \neq j$. The channel capacity is equal to

A. K bits per symbol.   B. $\log_2 K$ bits per symbol.   C. 0 bits per symbol.   D. $0.5K$ bits per symbol.

Q. 3. Median of a continuous random variable $\mathcal{R}$ is the solution of the equation $F_{\mathcal{R}}(r) = 0.5$, where $F_{\mathcal{R}}(r)$ denotes the cumulative distribution function (CDF). The probability density function $p_{\mathcal{R}}(r)$ of Rayleigh random variable $\mathcal{R}$ is given by

$$p_{\mathcal{R}}(r) = \begin{cases} re^{-\frac{r^2}{2}}, & 0 \leq r \leq \infty, \\ 0, & \text{otherwise.} \end{cases}$$

The median of Rayleigh distribution is approximately equal to

A. 1.7   B. 2.9   C. 0.7   D. None of the above

Q. 4. Suppose that a single-tone message signal is encoded by pulse code modulation. If the sampling frequency is 9 MHz, the minimum number of bits, and, corresponding bit rate required to achieve signal power-to-quantization noise power ratio greater than 45 dB are equal to ——- and, —— respectively.

A. 7, 63 Mbps   B. 8, 72 Mbps   C. 1, 9 Mbps   D. None of the above

Q. 5. Suppose that a signal $x(t) = e^{-\lambda t}, t > 0, \lambda > 0$ is applied as input to a filter whose transfer function is given by $H(f) = \frac{\mu}{\sqrt{\mu^2+4\pi^2 f^2}}, \mu > 0$. If the output energy is equal to half of the input energy, the relation between λ and μ is given by

A. $\lambda = 2\mu$ B. $\lambda = \frac{1}{\mu}$ C. $\lambda = \mu$ D. None of the above

Q. 6. Suppose that a given communication system uses binary phase shift keying (BPSK) modulation to transmit binary data at 54 Mbps in the presence of additive white Gaussian noise (AWGN) with power spectral density $N_0 = 1.67 \times 10^{-20}$ W/Hz. If the average transmit power is 9 dB and net system loss of 120 dB, the approximate bit error rate (BER) of the system in terms of $Q-$function is equal to ——

A. Q(2.1)   B. Q(4.2)   C. Q(1.05)   D. None of the above

Q. 7. Suppose that the outputs of a convolution encoder are given by $v_1 = s_1 \oplus s_2, v_2 = s_1 \oplus s_2 \oplus s_3, v_3 = s_1 \oplus s_3 \oplus s_4$, where $s_k, k = 1, 2, 3, 4,$ denote the binary state available at the output of each stage until it is shifted into the next stage. Input serial data is applied to the first stage. The code rate, and, the number of states in the state diagram, are equal to —— and,——respectively.

A. $\frac{1}{3}$, 3 B. $\frac{1}{4}$, 8 C. $\frac{1}{3}$, 16 D. None of the above

Q. 8. The four symbols B, I, T, S occur with probabilities 0.5, 0.25, 0.125, 0.125, respectively. Assuming the symbols are statistically independent, the information in the four-symbol message $I_M = BITS$ is equal to ———

A. 6 bits B. 9 bits C. 4 bits D. None of the above

Q. 9. Let Y_1, Y_2,…,Y_n denote i.i.d. exponential random variables with mean 1. The probability density function of $Z = \max(Y_1, Y_2, \ldots, Y_n)$ is given by

A. $n\exp(-nz)$ B. $n\left(1-\exp(-z)\right)^{n-1}\exp(-z)$ C. $\left(1-\exp(-z)\right)^{n}$ D. None of the above

Q. 10. Suppose that a 100 MHz carrier signal $c(t) = 10\cos(2\pi \times 10^8 t)$ is DSBSC modulated by a 1 MHz modulating signal $m(t) = \cos(2\pi \times 10^6 t)$. The output of the balanced modulator is passed through an ideal high pass filter with cut off frequency 100 MHz. A 100 MHz signal $d(t) = \sin(2\pi \times 10^8 t)$ is added at the output of the filter. The envelope of the resulting signal is ——-

A. $\sqrt{125 - 100\sin\left(2\pi \times 10^6 t\right)}$ B. $\sqrt{125 + 100\sin\left(2\pi \times 10^6 t\right)}$ C. $\sqrt{125 - 100\sin\left(2\pi \times 10^8 t\right)}$

D. None of the above

Q. 11. Consider a binary digital communication system which transmits 0's and 1's that are equally likely. When 0 is transmitted the voltage at the detector input can lie between -0.25 volt and +0.25 volt with equal probability. On the other hand, when 1 is transmitted, the voltage at the detector can have any value between 0 volt and 1 volt with equal probability. Suppose that the detector uses 0.2 volt threshold, the average probability of error is equal to ——— (Hint: Come up with uniform distributions to determine the error events. Then, find the area that contributes to error.)

A. 0.15 B. 0.20 C. 0.10 D. None of the above.

Q. 12. Consider coherent BPSK, binary amplitude shift keying (BASK), and, binary frequency shift keying (BFSK). Considering symbol error probability (SEP) performance for comparison, which of the following is true? Assume AWGN channel, and the same average bit energy in all the constellations.

A. $\text{SEP}|_{\text{BPSK}} > \text{SEP}|_{\text{BFSK}} > \text{SEP}|_{\text{BASK}}$ B. $\text{SEP}|_{\text{BPSK}} < \text{SEP}|_{\text{BFSK}} > \text{SEP}|_{\text{BASK}}$

C. $\text{SEP}|_{\text{BPSK}} < \text{SEP}|_{\text{BFSK}} < \text{SEP}|_{\text{BASK}}$ D. None of the above.

Q. 13. The SNR at the output of an AM receiver is 40 dB. The baseband signal is band-limited to 4 KHz. The carrier signal is 100% modulated by the single-tone modulating signal. The transmitted signal is affected by AWGN channel with power spectral density of 10^{-9} W/Hz. The maximum amplitude of the carrier is approximately equal to ——-

A. 0.9 volt B. 1.8 volt C. 1.8 millivolt D. None of the above.

Q. 14. Suppose that an angle modulated signal with carrier frequency $f_c = 10^6$ Hz is given by

$$s(t) = 10\cos\left(2\pi \times 10^6 t + 140\pi\sin(150t) + 480\pi\cos(150t)\right).$$

The maximum frequency deviation of the modulated signal is equal to ——

A. 75 KHz B. 7.5 KHz C. 37.5 KHz D. None of the above.

Q. 15. A stationary, zero-mean Gaussian random process $Y(t)$ has autocorrelation function $\mathcal{R}_Y(\tau)$. The variance of the process is estimated by forming the quantity

$$Z = \frac{1}{T}\int_0^T y^2(t)\,dt,$$

where $y(t)$ denotes a sample function of the process. The expected value of Z is equal to ——

A. One B. Zero C. $\mathcal{R}_Y(0)$ D. None of the above.

□ END OF PART-A □

Answers

1. D. *Discussion:* (ii) is False. Because, mean square value of noise current is directly proportional to the bandwidth.

2. B. *Discussion:* Recall that channel capacity is maximum mutual information. Since $H(X|Y)$ is zero for a noiseless channel, $I(X;Y) = H(X)$. Entropy is maximum when the symbols are equally likely. Therefore, $\mathcal{C} = \log_2 K$ bits per symbol.

3. D. *Discussion:* Median is the solution of the equation $1 - e^{-\frac{r^2}{2}} = 0.5$. Solving for r, we get 1.1774.

4. B. *Discussion:* Recall that SQNR $= 6.02n + 1.76$ dB. To achieve the SQNR greater than 45 dB, the minimum number of bits required is equal to 8 and corresponding bit rate is equal to 72 Mbps.

5. C. *Discussion:* Recall that area under energy spectral density gives energy. Therefore, we have

$$\int_{-\infty}^{\infty} S_Y(f)\,df = 0.5 \int_{-\infty}^{\infty} S_X(f)\,df.$$

Recall that $S_Y(f) = |H(f)|^2 S_X(f)$, where $S_X(f) = \frac{1}{\lambda^2 + 4\pi^2 f^2}$. Simplifying further, yields $\lambda = \mu$.

6. B. *Discussion:* The bit error rate is equal to $Q\left(\sqrt{\frac{2\times 7.94\times 10^{-12}}{1.67\times 10^{-20}\times 54\times 10^6}}\right) = Q(4.1963)$.

7. C. *Discussion:* Since one input bit is encoded to three output bits, the code rate is equal to $\frac{1}{3}$. Furthermore, since there are four stages, the size of state space is $2^4 = 16$.

8. B. *Discussion:* Since the symbols are independent, the measure of information is additive. Therefore, we have $I_M = 1 + 2 + 3 + 3 = 9$ bits.

9. B. *Discussion:* First find CDF. The CDF is given by $\mathcal{P}(Z \le z) = \mathcal{P}(\max(Y_1, Y_2, \ldots, Y_n) \le z) = (1 - e^{-z})^n$. By differentiating the CDF, we get the PDF.

10. A. *Discussion:* Output of the balanced modulator is given by

$$s(t) = \frac{A_c A_m}{2} \cos\left(2\pi(f_c + f_m)t\right) + \frac{A_c A_m}{2} \cos\left(2\pi(f_c - f_m)t\right),$$

where $A_c = 10$ volt, $A_m = 1$ volt, $f_c = 100$ MHz, and, $f_m = 1$ MHz. The output of HPF is given by $s(t) = \frac{A_c A_m}{2} \cos\left(2\pi(f_c + f_m)t\right)$. Since $d(t) = \sin(2\pi \times 10^8 t)$ is added, the resultant signal becomes $v(t) = \frac{A_c A_m}{2} \cos\left(2\pi(f_c + f_m)t\right) + A_c \sin(2\pi f_c t)$. Express $v(t)$ in canonical form and find envelope.

11. A. *Discussion:* Average probability of error $= \frac{1}{2}\left(p\,(e|0) + p\,(e|1)\right)$. Identify the error regions. Verify that $p\,(e|0) = 0.1$ and $p\,(e|1) = 0.2$. Therefore, the average error probability is equal to 0.15.

12. C. *Discussion:* This can be easily shown from the fact that $d_{\min}|_{\text{BPSK}} > d_{\min}|_{\text{BFSK}} > d_{\min}|_{\text{BASK}}$.

13. D. *Discussion:* Recall that the figure of merit of AM in AWGN is given by

$$\text{FOM} = \frac{\text{SNR}_0}{\text{SNR}_i} = \frac{1}{3}.$$

Therefore, $\text{SNR}_i = 3 \times 10^4$. Since noise power $n_i = N_0 \times W$, the transmitted signal power $\frac{A_C^2}{2}\left(1 + 0.5 \times \mu^2\right) = 0.24$. Since $\mu = 1$, we get $A_c \approx 0.57$ volt.

14. C. *Discussion:* Instantaneous angular frequency of the given angle modulated signal is given by

$$\begin{aligned}\omega_i &= \frac{d\theta_i(t)}{dt},\\ &= 2\pi \times 10^6 t + 3000\pi\left(7\cos(150t) - 24\sin(150t)\right),\\ &= 2\pi \times 10^6 t + 75000\pi \cos\left(150t + \delta\right),\end{aligned}$$

where $\delta = \tan^{-1}\left(\frac{24}{7}\right)$. Angular frequency deviation $\Delta\omega = 75000\pi \cos\left(150t + \delta\right) \Rightarrow \Delta\omega_{\max}$ is equal to 75000π. Therefore, the maximum frequency deviation is equal to 37.5 KHz.

15. C. *Discussion:* The expected value of Z is given by

$$\begin{aligned}
\mathbf{E}\left[Z\right] &= \mathbf{E}\left[\frac{1}{T}\int_0^T y^2(t)\,dt\right], \\
&= \frac{1}{T}\int_0^T \mathbf{E}\left[y^2(t)\right]\,dt, \\
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Answers

1. D. *Discussion:* Recall that the figure of merit of AM in AWGN is given by

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where $A_c = 10$ volt, $A_m = 1$ volt, $f_c = 100$ MHz, and, $f_m = 1$ MHz. The output of HPF is given by $s(t) = \frac{A_c A_m}{2}\cos\left(2\pi(f_c + f_m)t\right)$. Since $d(t) = \sin(2\pi \times 10^8 t)$ is added, the resultant signal becomes $v(t) = \frac{A_c A_m}{2}\cos\left(2\pi(f_c + f_m)t\right) + A_c\sin(2\pi f_c t)$. Express $v(t)$ in canonical form and find envelope.

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15. B. *Discussion:* Recall that channel capacity is maximum mutual information. Since $H(X|Y)$ is zero for a noiseless channel, $I(X;Y) = H(X)$. Entropy is maximum when the symbols are equally likely. Therefore, $\mathcal{C} = \log_2 K$ bits per symbol.

COMPREHENSIVE EXAMINATION: PART-B

Instructor-in-Charge: B. Sainath, 2210-A, Electrical and Electronics Eng. Dept., BITS Pilani.
Course No./Title : EEE F311/Communication Systems DATE: 7th Dec. 2017 (FN)
Max. marks: 50 Max. Time: 120 Mins.

Important instructions:

- This part of the test is open book. Mention this on the top of your answer sheet.
- Write your ID number, name, tutorial section number, and instructor's name on your answer sheet.
- You can answer questions in any order. But, provide answers to all subparts within a question at one place. Give all your answers with proper units and do not skip intermediate steps.
- In all sketches, include x-label, y-label, figure caption. Incomplete figures will not get full credit.
- Over-written answers will not be rechecked.

Q. 1. [Huffman Coding]

An information source $\mathcal{S}$ delivers seven distinct symbols $s_1, \ldots, s_7$ with probabilities $\frac{7}{20}$, $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{25}$, $\frac{1}{200}$, $\frac{1}{200}$ respectively. Answer the following.

i). Using Huffman's source coding algorithm, come up with the code set for the seven symbols. *Note:* Use the algorithm discussed in class. [7 marks]

ii). Compute the following:

a). Entropy of the source $\mathcal{S}$. [1 mark] b). The average code word length $\overline{L}$. [1 mark]

c). Compute code efficiency and redundancy. [1 mark]

Q. 2. [Linear Block Code & Its Error Performance]

For a (7, 4) systematic linear block, the three parity check bots p_5, p_6, and p_7 are obtained from the following.

$$p_5 = d_1 \oplus d_3 \oplus d_4,$$
$$p_6 = d_1 \oplus d_2 \oplus d_3,$$
$$p_7 = d_2 \oplus d_3 \oplus d_4.$$

Answer the following. *Note:* Your approach should be consistent with the one discussed in class.

i). Write down the generator matrix G, and, the transpose of the parity check matrix, denoted by H^T. [1 + 1 marks]

ii). Determine code words corresponding $\{1000, \ldots, 1111\}$. Present them in tabular form. [4 marks]

iii). Suppose that the received code word is 0010110. Determine the syndrome and decode the code word by syndrome decoding. [1 + 1 marks]

iv). Suppose that the bit energy-to-noise power spectral density ratio ($\frac{E_b}{N_0}$) is equal to 10 dB and coherent phase shift keying is used for information transmission. Compare the word error probability performance of an additive white Gaussian noise (AWGN) binary symmetric channel (BSC) using (7, 4) code with that of same system using uncoded information transmission. Note that the minimum Hamming distance of the linear block code is 3. Assume that the bit error probability in the uncoded case, as well as, in the coded case is much less than 1. [2 marks]

Q. 3. [$4-$PAM Symbol Error Probability (SEP)]

Consider transmission of symbols drawn from $4-$PAM constellation shown in Figure 1 through AWGN channel. Assume that all symbols are equally likely.

i). If the average transmitted energy per symbol is 1 joule, determine the value of b. [2 marks]

ii). Show the decision regions and label them in a sequence from left to right. [2 marks]

iii). Derive average symbol error probability in terms of complimentary error function. [4 + 1 marks]

iv). Derive an upper bound for the SEP. [1 mark]

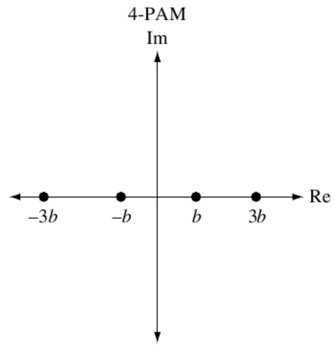

Fig. 1: 4-PAM constellation diagram

Q. 4. [Low-pass RC Filter as Approximate Matched Filter]

Consider a rectangular pulse $s(t)$ defined by

$$s(t) = \begin{cases} A, & 0 \leq t \leq T, \\ 0, & \text{otherwise.} \end{cases}$$

Suppose that we need to approximate the matched filter by a low-pass RC filter characterized by

$$H(f) = \frac{1}{1 + j\frac{f}{f_0}},$$

where $f_0 = \frac{1}{2\pi RC}$ is the 3 dB bandwidth of the filter.

Answer the following.

i). Sketch the output of the filter. At what value of time t, peak occurs? Indicate the peak value. [2 marks]

ii). Determine the maximum value of the output pulse power. [1 mark]

iii). Determine the mean value of the output noise power. Assume AWGN channel. [1 mark]

iv). Determine the output signal power-to-noise power ratio (SNR). What is its maximum value? (Hint: Differentiate with respect to $f_0 T$.). At what value of f_0, maximum output SNR occurs? [1 + 2 + 1 marks]

v). By how many dB should the transmitted pulse energy be increased so as to achieve the same performance as the perfectly matched filter? [2 marks]

Q. 5. [Parameter Estimation]

Consider a finite-duration signal of the form

$$s(t, A) = \begin{cases} As(t), & 0 \leq t \leq T, \\ 0, & \text{otherwise,} \end{cases}$$

where $s(t)$ is completely known and amplitude A is unknown. Recall that the maximum likelihood estimate (MLE) $\hat{A}$ is the solution of

$$\int_0^T [x(t) - s(t, \hat{A})] \frac{\partial s(t, \hat{A})}{\partial \hat{A}} \, dt = 0,$$

where $x(t) = As(t) + w(t)$ denotes the received signal. Note that $w(t)$ denotes AWGN.

i). Determine the ML estimate of A in the presence of AWGN. [2 marks]

ii). Comment on the implementation of $\hat{A}$ and come up with a scheme to determine the MLE. [3 marks]

iii). Determine the mean and variance of the estimate. [2 + 3 marks]

□ **END OF PART-B** □

Answers

Q. 1. i). Huffmann code = $\{1, 01, 000, 0010, 00110, 001110, 001111\}$.

ii). a). Entropy $H(\mathcal{S}) = 2.11$ bits/symbol.

b). Average codeword length $\overline{L} = 2.21$ bits/symbol.

c). Code efficiency $\eta = \frac{H(\mathcal{S}}{\overline{L}} = 95.48\%$ and redundancy 4.52%.

Q. 2. i). In the given error correcting code, $n = 7$ and $k = 4$. We have $P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Generator matrix $G = [I_4; P^T]$, where I_3 denotes 3×3 identity matrix, and P^T denotes the transpose of the matrix P. So,

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

The transpose of the parity-check matrix is given by $H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

ii). The table is shown below.

Message word	Code word
1 0 0 0	1 0 0 0 1 1 0
1 0 0 1	1 0 0 1 0 1 1
1 0 1 0	1 0 1 0 0 0 1
1 0 1 1	1 0 1 1 1 0 0
1 1 0 0	1 1 0 0 1 0 1
1 1 0 1	1 1 0 1 0 0 0
1 1 1 0	1 1 1 0 0 1 0
1 1 1 1	1 1 1 1 1 1 1

iii). Since d_{min} is 3, the code is a single-error-correcting code. So, t is equal to 1. Let r denotes the received code word. The syndrome is given by $s = rH^T = [0\ 0\ 1]$. Since s is equal to the seventh row of H^T, an error is in seventh bit. So the code word should be 0010111. So, the data bits are 0010.

iv). Word error probability in the uncoded case $\approx kQ\left(\sqrt{\frac{2E_b}{N_0}}\right) = 1.5488 \times 10^{-5}$. In the coded case, word error probability is $\approx \binom{n}{t+1}\left(Q\left(\sqrt{\frac{2kE_b}{nN_0}}\right)\right)^{t+1} = 2.7461 \times 10^{-6}$.

Q. 3. i). Average energy per symbol, $\mathcal{E}_s = 5b^2$. If $\mathcal{E}_s = 1$, $b = \sqrt{\frac{1}{5}}$.

ii). Draw perpendicular bisectors for each segment joining two constellation points and identify the decision regions. Let the decision regions (from left to right) denoted by $\Delta_1 \ldots \Delta_4$. These are shown in the Figure 2.

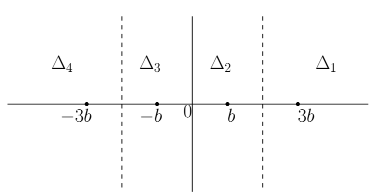

Fig. 2: Decision regions of symbols in the 4-PAM constellation.

iii). Let us now compute probability of error corresponding to each transmitted symbol.

$x = -3b$ transmitted: Let y denote the received symbol.

$$\begin{aligned}\mathcal{P}(\mathcal{E}|x=-3b) &= 1-\mathcal{P}(\mathcal{C}|x=-3b),\\ &= 1-\mathcal{P}(y<-2b|x=-3b),\\ &= Q\left(\sqrt{\frac{2b^2}{N_0}}\right),\end{aligned}$$

where $\frac{N_0}{2}$ is the power spectral density of additive Gaussian noise, Note that, by symmetry, $\mathcal{P}(\mathcal{E}|x=-3b) = \mathcal{P}(\mathcal{E}|x=3b)$.

Similarly, we can easily compute error probabilities for other symbols:

$$\begin{aligned}\mathcal{P}(\mathcal{E}|x=-b) &= \mathcal{P}(\mathcal{E}|x=b),\\ &= 2Q\left(\sqrt{\frac{2b^2}{N_0}}\right).\end{aligned}$$

Therefore, the average symbol error probability (P_e or SEP) is given by $\frac{3}{2}Q\left(\sqrt{\frac{2}{5N_0}}\right)$. Since $Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right)$, Therefore, SEP $= \frac{3}{4}\text{erfc}\left(\sqrt{\frac{1}{5N_0}}\right)$.

iv). Recall that $Q(y) \le \frac{1}{2}e^{-\frac{y^2}{2}}$. Using this inequality, we get $\frac{3}{2}Q\left(\sqrt{\frac{2}{5N_0}}\right) \le \frac{3}{4}e^{-\frac{1}{5N_0}}$.

Q. 4. i). The impulse response of the RC low-pass filter is given by

$$h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}, t > 0$$

Since $f_0 = \frac{1}{2\pi RC}$, $RC = 2\pi f_0 \triangleq b$.

Output $y(t)$ of the filter is the convolution of the pulse $s(t)$ and the impulse response $h(t)$. It can be shown that

$$y(t) = \begin{cases} 0, & t < 0,\\ A\left(1-e^{-bt}\right), & 0 \le t \le T,\\ A\left(e^{-b(t-T)} - e^{-bt}\right), & t \ge T.\end{cases}$$

The output $y(t)$ is shown in the Figure 3.

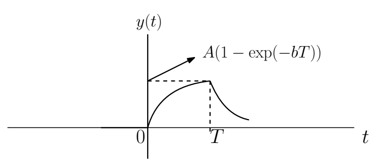

Fig. 3: Output of the RC low-pass filter.

Peak occurs at $t = T$ and the maximum value is equal to $A\left(1-e^{-2\pi f_0 T}\right)$.

ii). The maximum value of the pulse power is equal to $A^2\left(1-e^{-2\pi f_0 T}\right)^2$.

iii). The mean value of the output noise power is given by

$$\begin{aligned}n_0 &= \frac{N_0}{2}\int_{-\infty}^{\infty}\frac{1}{1+\left(\frac{f}{f_0}\right)^2}df,\\ &= \frac{\pi N_0 f_0}{2}.\end{aligned}$$

iv). Output SNR is equal to $\text{SNR}_0 = \frac{2A^2}{\pi N_0 f_0}\left(1-e^{-2\pi f_0 T}\right)^2$. The maximum output SNR is equal to $\text{SNR}_{0,\max} = \frac{1.62A^2T}{N_0}$. The maximum SNR occurs at $f_0 = \frac{0.2}{T}$.

v). Energy of the pulse, denoted by $E = A^2T$. For a perfectly matched filter, the maximum output SNR should be equal to $\frac{2E}{N_0} = \frac{2A^2T}{N_0}$. Therefore, the transmitted energy should be increased by the ratio $\frac{2}{1.62}$ i.e. by approximately 0.92 dB so that the low-pass RC filter with $f_0 = \frac{0.2}{T}$ realizes the same performance of a perfectly matched filter.

Q. 5. i). The MLE $\hat{A}$ of the amplitude A is the solution of the equation:

$$\int_0^T [x(t) - \hat{A}s(t)]s(t)\,dt = 0,$$

Solving for $\hat{A}$, we get

$$\hat{A} = \frac{\int_0^T x(t)s(t)\,dt}{\int_0^T s^2(t)\,dt}.$$

ii). *Comments or Remarks:* The operations may be performed using correlator, squaring device, integrator, and divider as shown in Figure 4.

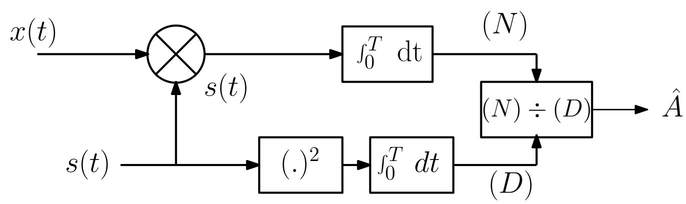

Fig. 4: MLE of amplitude in AWGN.

iii). Note that $w(t)$ has zero mean. Therefore, the mean value of the estimate is given by

$$\begin{aligned}\mathbf{E}\left[\hat{A}\right] &= \frac{\int_0^T As^2(t)\,dt}{\int_0^T s^2(t)\,dt},\\ &= A.\end{aligned}$$

Let $\mathcal{E} = \int_0^T s^2(t)\,dt$. The variance of the estimate is given by

$$\begin{aligned}\mathrm{Var}\hat{A} &= \mathrm{Var}\left(\frac{\int_0^T x(t)s(t)\,dt}{\mathcal{E}}\right),\\ &= \frac{1}{\mathcal{E}^2}\mathrm{Var}\left(\int_0^T w(t)s(t)\,dt\right),\\ &= \frac{N_0}{2\mathcal{E}}.\end{aligned}$$