BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
SECOND SEMESTER 2016-2017
EEE F416 DIGITAL COMMUNICATION
COMPREHENSIVE EXAMINATION [CLOSED-BOOK]
May 10, 2017

MAX. MARKS: 80 **Time: 90 min.**

Note:- All questions carry equal marks. State valid assumptions, if any, clearly

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1. Determine and sketch the autocorrelation function of a Gaussian pulse defined by $g(t) = \frac{1}{t_0}\exp\left(-\frac{\pi t^2}{t_0^2}\right)$

2. Prove the following two properties of the autocorrelation function $R_X(\tau)$ of a random process $X(t)$: (a) If $X(t)$ contains a dc component equal to A, then $R_X(\tau)$ contains a constant component equal to A^2; (b) If $X(t)$ contains a sinusoidal component, then $R_X(\tau)$ also contains a sinusoidal component of the same frequency.

3. A telegraph system transmits either a dot or dash signal. Assume the transmission properties are such that 2/5 of the dots and 1/3 of the dashes are received incorrectly. Suppose the ratio of transmitted dots to transmitted dashes is 5 to 3. What is the probability that a received signal is as transmitted if: a) the received signal is a dot? b) The received signal is a dash?

4. In a computer-communication network, the arrival time τ between messages is modeled with an exponential distribution function, having the density,

$$f_T(\tau) = \begin{cases} \frac{1}{\lambda} e^{-\frac{\tau}{\lambda}} & \tau > 0 \\ 0 & o.w. \end{cases}$$

a) What is the mean time between messages with this distribution? b) What is the variance in this time between messages?

5. A PAM telemetry system involves the multiplexing of four input signals: $s_i(t)$, i = 1,2,3,4. Two of the signals $s_1(t)$ and $s_2(t)$ have bandwidths of 80 Hz each, whereas the remaining two signals $s_3(t)$ and $s_4(t)$ have bandwidths of 1 kHz each. The signals $s_3(t)$and $s_4(t)$ are each sampled at the rate of 2400 samples per second. This sampling rate is divided by 2^R (i.e., an integer power of 2) in order to derive the sampling rate for $s_1(t)$ and $s_2(t)$. (a) Find the maximum value of R. (b) Using the value of R found in part (a), design a multiplexing system that first multiplexes $s_1(t)$ and $s_2(t)$ into a new sequence, and then multiplexes $s_3(t)$, $s_4(t)$ and $s_5(t)$.

6. An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 128. A synchronizing pulse is added at the end of each code word. The resulting PCM signal is transmitted over a channel of bandwidth 13 kHz using a quaternary PAM system with a raised-cosine pulse spectrum. The roll-off factor is unity.

(a) Find the rate (in bits per second) at which information is transmitted through the channel.

(b) Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal?

7. Consider an MSK modulator that uses a sinusoidal carrier with frequency f_c = 50 MHz. The bit rate of the incoming binary stream is 20 kbps. (a) Calculate the instantaneous frequency of the MSK modulator for a data sequence in which symbols 0 and 1 alternate. (b) Repeat the calculation in part (a) for a data sequence that consists of all 0s. What if the sequence consists of all 1s?

8. Consider a phase-locked loop consisting of a multiplier, loop filter, and voltage-controlled oscillator (VCO). Let the signal applied to the multiplier input be a PSK signal defined by $s(t) = A_c \cos[2\pi f_c t + k_p b(t)]$, where, k_p is the phase sensitivity, and the data signal $b(t)$ takes on the value +1 for binary symbol 1 and -1 for binary symbol 0. The VCO output is $r(t) = A_c \sin[2\pi f_c t + \theta(t)]$. (a) Evaluate the loop filter output, assuming that this filter removes only modulated components with twice the carrier frequency f_c. (b) Show that this output is proportional to the data signal $b(t)$ when the loop is phase locked; that is, $\theta(t) = 0$.

9. A binary FSK system transmits data at the rate of 2.5 megabits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} watts per hertz is added to the signal. In the absence of noise, the amplitude of the received signal is 1 μV across 50 ohm impedance. Determine the average probability of error assuming coherent detection of the binary FSK signal.

10. Show that the noise variance of the in-phase component $n_I(t)$ of the band-pass noise is the same as the band-pass noise $n(t)$ variance; that is, for a band-pass noise bandwidth B_N,

$$E[n_I^2(t)] = N_0 B_N$$
