Birla Institute of Technology and Science Pilani (Rajasthan)

EEE F416 Digital Communication
Second Semester 2022-23
Date : 15/03/2023 **Mid Semester Exam** **Max. Marks: 60**

Name: ________________________________ID No. ________________ **Time: 90 min**

Instructions:

1) This is a closed book, closed notes exam. Only 1 A4 size, double sided hand-written formula sheet is allowed.

2) Show all the steps clearly. If I cannot interpret it, I cannot grade it.

Q.1a) Consider a set of M orthogonal signal waveforms $s_m(t)$, for $1 \leq m \leq M$ and $0 \leq t \leq T$, where each waveform has the same energy E. A new set of M waveforms are defined as, $s'_m(t) = s_m(t) - \frac{1}{M}\sum_{i=1}^{M} s_i(t)$, $1 \leq m \leq M$ and $0 \leq t \leq T$. Find the energy of $s'_m(t)$, and the inner product between any two waveforms in the new set in terms of E and M. [6]

Q.1b) Using the Gram-Schmidt orthogonalization procedure, find a set of orthonormal basis functions to represent the three signals $s_1(t), s_2(t)$, and $s_3(t)$ as shown in Fig.1. Express each of these signals in terms of the set of basis functions. [6]

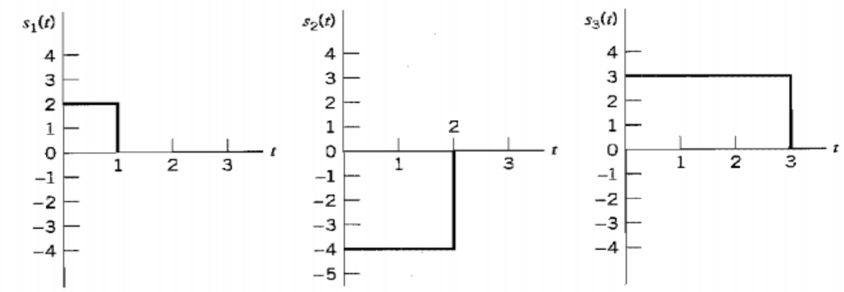

Fig. 1

Q.2a) A pair of noise processes $n_1(t)$ and $n_2(t)$ are related by $n_2(t) = n_1(t)\cos(2\pi f_c t + \theta) - n_1(t)\sin(2\pi f_c t + \theta)$, where f_c is constant, and θ is the value of random variable Θ whose probability density function is defined by $f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta \leq 2\pi \\ 0, & otherwise \end{cases}$. The noise process $n_1(t)$ is stationary and its power spectral density is shown in Fig. 2. Find and plot the corresponding power spectral density of $n_2(t)$. [8]

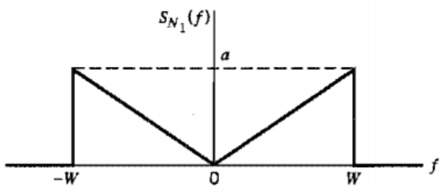

Fig. 2

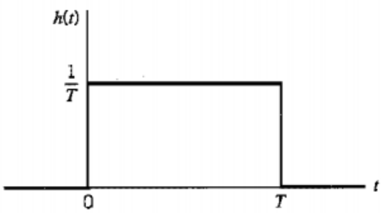

Fig. 3

Q.2b) A stationary Gaussian process $X(t)$ with zero mean and power spectral density $S_X(f)$ is applied to a linear filter whose impulse response $h(t)$ is shown in Fig. 3. A sample Y is taken of the random process at the filter output at time T. Find the mean, variance and probability density function of Y. [7]

P.T.O

Q.3) A ternary communication system transmits one of the three equiprobable signals $s(t), 0, -s(t)$ every T seconds. Correspondingly, the received signal is $r(t) = s(t) + z(t),\ r(t) = z(t),$ or $r(t) = -s(t) + z(t)$, where $z(t)$ is white Gaussian noise with $E[z(t)] = 0$ and $R_z(\tau) = E[z(t)z^*(\tau)] = 2N_o\delta(t-\tau)$. The optimum receiver computes the correlation metric $C = Re[\int_0^T r(t)s^*(t)dt]$ and compares it with a threshold A and a threshold $-A$. If $C > A$, the decision is made that $s(t)$ was sent. If $C < -A$, the decision is made in favor of $-s(t)$. If $-A < C < A$, the decision is made in favor of 0.

a) Determine the three conditional probabilities of error: P_e given that $s(t)$ was sent, P_e given that $-s(t)$ was sent, and P_e given that 0 was sent. [6]

b) Determine the average probability of error $P_{e(avg)}$ as a function of threshold A. [2]

c) Determine the value of A that minimizes $P_{e(avg)}$. [4]

Q.4) Fig.4(a) shows a pair of pulses that are orthogonal to each other over the interval $[0, T]$.

a) Determine the matched filters and sketch their responses for pulses $s_1(t)$ and $s_2(t)$ considered individually. [3]

b) Form a two-dimensional matched filter by connecting the two matched filters of Part(a) in parallel, as shown in Fig. 4(b).

(i) Sketch the lower matched filter output, and determine its value at $t = T$ when the pulse $s_1(t)$ is applied to the two-dimensional filter. [6]

(ii) Sketch the upper matched filter output, and determine its value at $t = T$ when the pulse $s_2(t)$ is applied to the two-dimensional filter. [6]

c) Prove that if a signal $s(t)$ is corrupted by AWGN, the filter response matched to $s(t)$ maximizes the output SNR. [6]

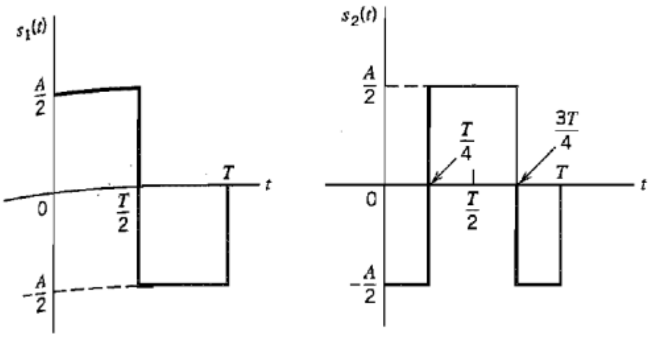

Fig. 4(a)

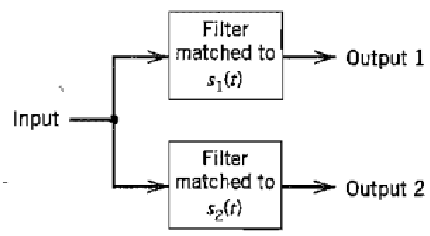

Fig. 4(b)

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