Birla Institute of Technology and Science Pilani (Rajasthan)

EEE F416 Digital Communication
Second Semester 2022-23

Date : 12/05/2023 **Comprehensive Exam** **Max. Marks: 80**

Name: _____ID No. _____ **Time: 180 min**

Instructions:

1) This is a closed book, closed notes exam. Only 2 A4 size, double sided hand-written formula sheet is allowed.

2) Show all the steps clearly. If I cannot interpret it, I cannot grade it.

Q.1a) There are 6 bottles of wine, out of which one bottle has gone bad (tastes terrible). From inspection of the bottles it is determined that the probability p_i of *i-th* bottle being bad is given by $(p_1, p_2, p_3, p_4, p_5, p_6) = (8/23, 6/23, 4/23, 2/23, 2/23, 1/23)$. Tasting will determine the bad wine. Suppose you taste the wines one at a time. Choose the order of tasting to minimize the expected number of tastings required to determine the bad bottle. Remember, if the first 5 wines pass the test you don't have to taste the last. **[2+1+4+1 = 8]**

(i) What is the expected number of tastings required?

(ii) Which bottle should be tasted first*?*

Now you get smart. For the first sample, you mix two of the wines in a fresh glass and sample the mixture. You proceed, mixing and tasting, stopping when the bad bottle has been determined.

(iii) What is the minimum expected number of tastings required to determine the bad wine?

(iv) What mixture should be tasted first?

Q.1b) Consider the Lempel Ziv encoding for quaternary data (symbols: 0,1,2,3). Encode the following data: 1 3 3 0 0 2 0 2 1 1 1 3 0 0 0 0 2 2 1 2 2 2 3 3. What is the compression ratio obtained? Does this scheme actually cause any compression? Justify. **[7]**

Q.2a) Find the optimum threshold γ_o for detecting equally likely signals, $s_1(t) = \sqrt{\frac{2E}{T}}\cos(\omega t), s_2(t) = \sqrt{\frac{E}{2T}}\cos(\omega t + \pi)$, in AWGN using a correlator receiver. Assume the basis signal $\psi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega t)$. **[5]**

Q.2b) A system using matched filter detection of equally likely BPSK signals, $s_1(t) = \sqrt{\frac{2E}{T}}\cos(\omega t), s_2(t) = \sqrt{\frac{2E}{T}}\cos(\omega t + \pi)$, operates in AWGN with a received $\frac{E_b}{N_o}$ of 6.8 dB. Let $z(t)$ be the output of the matched filter. Assume that $\boldsymbol{E}\{z(T)\} = \pm\sqrt{E}$.

(i) If the decision threshold is $\gamma = 0.1\sqrt{E}$, find the bit error probability P_b. **[10]**

(ii) The threshold of $\gamma = 0.1\sqrt{E}$ is optimum for a particular set of priori probabilities $P(s_1)$ and $P(s_2)$. Find the values of these probabilities. **[5]**

Q.3a) Explain the Nyquist pulse shaping criterion for bandlimited communication and its significance**.** Prove the condition of Nyquist pulse shaping in frequency domain. **[10]**

Q.3b) The transmission of a signal pulse with a raise cosine spectrum through a channel results in the following (noise-free) sampled output from the demodulator: $x_k = \{-0.5, 0.1, 1, -0.2, 0.05, 0\}$ for $k = \{-2, -1, 0, 1, 2, otherwise\}$, respectively. Assume that the pulse is sampled at $1/T$, which is also the symbol rate. **[8+7 = 15]**

(i) Determine the tap coefficients of a three-tap linear equalizer based on the zero-forcing criterion.

(ii) For the coefficients determined in (i), determine the output of the equalizer. Thus, determine the residual ISI and its span in time.

P.T.O

Q.4a) In a coherent FSK system, the signals $s_1(t)$ and $s_2(t)$ representing symbols 0 and 1, respectively are defined by,

$$s_1(t), s_2(t) = A_c \cos\left[2\pi\left(f_c \pm \frac{\Delta f}{2}\right)t\right], \quad 0 \le t \le T_b$$

Assuming that $f_c > \Delta f$, find the correlation coefficient of signals $s_1(t)$ and $s_2(t)$, and hence determine the minimum value of frequency shift Δf for which $s_1(t)$ and $s_2(t)$ are orthogonal. **[10]**

Q.4b) White Gaussian noise of zero mean and power spectral density $N_o/2$ is applied to the filtering scheme as shown in Fig. 1a. The frequency responses of these two filters are shown in Fig. 1b. The noise at the low pass filter output is denoted by $n(t)$. **[7+3=10]**

(i) Find the power spectral density and the autocorrelation function of $n(t)$.

(ii) Find the mean and variance of $n(t)$.

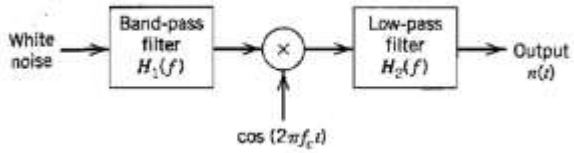

Fig. 1a

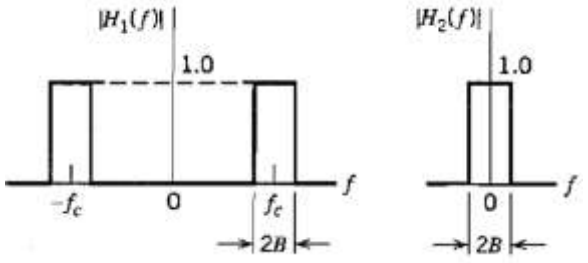

Fig. 1b

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