## Duration: 180 minutes General Instructions:

Max. Marks: 80

## 1. All questions are mandatory.

2. Write concise answers to the point for both theoretical and numerical questions. Do not write an essay that is irrelevant to what is asked in the theory questions. For numerical questions, mention important steps and do not write the final answer directly.

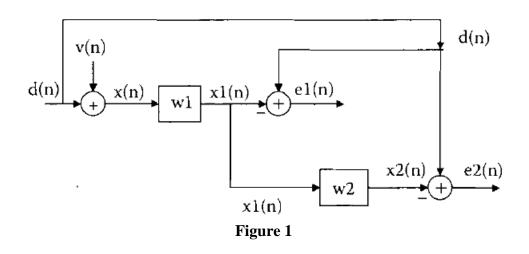
**Q1a.** A Gaussian random variable x(n) has mean  $\mu_n$  and variance  $\sigma_n^2$ , with  $\sigma_n^2 > 0$ . Find the mean and variance of  $w(n) = (x(n) - \mu_n)/\sigma_n^2$ . [5]

**Q1b.** Determine the auto-correlation of  $x(n) = \sum_{k=1}^{M} A_k \cos(\omega_k n + \varphi_k), \ \omega_k \neq 0$  [5]

## Q2

- a) The pattern observed in the phonocardiogram signal is S1S2S1S2S1S1S2. What is your observation from looking at this pattern.
- **b**) An EEG signal (s(t)) is recorded from a subject. A signal  $p(t) = s(t)\cos(4\pi t)$  is observed. The spectrum of p(t) has only one peak at f=5 Hz. What the subject is doing when s(t) is recorded from his/her brain.
- c) Which biomedical signal is best to track the eye movement of a subject?
- d) The relation between two random variables is  $y^2 + x^2 = 4$ . What is the correlation coefficient between them?
- e) What is multiresolution analysis?

**Q3** The block diagram shown in Figure 1 consists of two causal Wiener filters. Both filters can be modelled as AP (1) system. The desire signal is d(n) = u(n). The signal x(n) = d(n) + v(n) is the input to the first Wiener filter, w1. The v(n) is the white noise with PSD equal to one. Assuming that the Wiener Hopf equation can be applied to AP model and the Z transform of the white noise is zero, determine the filter coefficients of filter, w2.



[15]

[1×5]

Q5 The frequency response of an LTI system H(z) = z. The Z transform of the input to this LTI system is  $X(z) = e^{z^2}$ . Determine the expression of cepstrum output  $\hat{y}(n)$ . If the Z transform of 2 cos ( $\hat{y}(n-1)$ ) is  $AZ^B + CZ^D$ , determine A, B, C, and D. [10]

Q6 Determine the slope of the power spectral density of PZ (P, Q) model with real coefficients at  $\omega = 0$  and  $\omega = \pi$ . [Note: P and Q are any positive integer value.] [10]

## Q7

- a) A white noise with PSD equal to 2 is fed to a system 1 which is an AP LTI system with impulse response  $h(n) = a^n u(n)$ . The output of the system 1 is y(n) with power spectral density  $S_{\nu}(e^{j\omega})$ . A system 2 is design to produce  $\hat{y}(n)$  which predicts the output y(n). The square of the Gain of system 1 is equal to the total square error between y(n) and  $\hat{y}(n)$ . The  $S_{\hat{y}}(e^{j\omega}) = [S_y(e^{j\omega})]^2$ . Find the value of a. [Hint: The PSD of output = PSD of input  $\times$  (system frequency response)<sup>2</sup>] [5]
- **b**) The PSD of the outer ear model has peaks at  $\omega = \pm \frac{\pi}{2}$  and zero at  $\omega = 0$ . Similarly, the PSD of middle ear model has peaks at  $\omega = \pm \frac{\pi}{3}$ . Determine the frequency response of [3] the overall model. [2]
- c) Explain the structure of Cochlea briefly.

Q8 A neural network is design which has an inner layer (Layer 0), a hidden layer (Layer 1), and an output layer (Layer 2). The number of nodes (N) in Layer 0, Layer 1, and Layer 2 are 4, 3, and 2, respectively. A node in the  $m^{th}$  layer (m = 0, 1, 2) and at  $n^{th}$  position is represented as  $a_n^{(m)}$ . The weight between  $a_j^{(m-1)}$  and  $a_i^{(m)}$  is represented as  $w_{ij}$ , where  $0 \le i, j \le N$ . The  $w_{ij} = 1$  if  $i \ge j$  and  $w_{ij} = -1$  when i < j. Some node values are  $a_0^{(0)} = 0$ ,  $a_1^{(0)} = 1$ ,  $a_2^{(0)} = 0$ 1,  $a_3^{(0)} = 0$ ,  $a_1^{(1)} = 0.5$ . The bias used at Layer 0 is  $b_0$  and Layer 1 is  $b_1 = 0.5$ . To normalize the value of each node, a sigmoid function  $\sigma(x) = \frac{1}{1+e^{-x}}$  is used. Find the value of nodes in [10] Layer 2.