

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
Modern Control Systems (EEE F422) [1st Semester, 2021-2022]
Mid-Semester Test: Part-A (Closed Book)

Max Time: 45 min

Max Marks: 30

Date: 04/11/22

Q1. (i) What are the dimensions of the controllability and observability matrices of a 4th order systems with three inputs and two outputs? [2]

(ii) What is the advantage of Modal Residualization method over Modal Truncation method for model order reduction of LTI systems? [1]

(ii) The natural dynamics of a 2nd order LTI system is given by [5]

$$\dot{\underline{x}} = \begin{bmatrix} -0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix} \underline{x}$$

Determine $\underline{x}(t)$ for an initial condition of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(iv) Convert the following state space representation to a diagonal representation. [5]

$$\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = [1 \quad 1] \underline{x}$$

(v) For a SISO system, show that the controllability property remains invariant under similarity transformation. [2]

Q2. (i) Determine the sign definiteness of the function [2]

$$f(x_1, x_2, x_3) = x_1^2 + 6x_1x_2 + 4x_2x_3 + 2x_2^2 + 3x_3^4$$

(ii) For stability analysis of linear systems using Lyapunov's direct method, we solve the Lyapunov equation $A^T P + PA = -Q$. However, this equation is valid for continuous time systems. Derive a similar equation for discrete time systems. [5.5]

(iii) Limit cycles are created or destroyed across the bifurcation point in transcritical bifurcation. True/False? [1]

(iv) Define describing function. What are the main assumptions for the success of the describing function method? [2]

(v) Determine the equilibrium points of the following nonlinear system and comment on the stability of the linearized system about the equilibrium points. [4.5]

$$\dot{x}_1 = x_1^2 - x_1x_2$$

$$\dot{x}_2 = x_1 - x_1^3$$

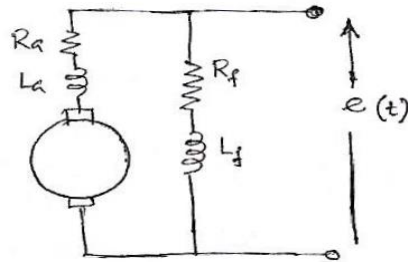
BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
Modern Control Systems (EEE F422) [1st Semester, 2022-2023]
Mid-Semester Test: Part-B (Open Book)

Max Time: 45 min

Max Marks: 30

Date: 04/11/22

Q1. Derive a state space model for a DC Shunt connected motor whose schematic diagram is shown below. Consider the applied voltage $e(t)$ and the load torque $T_L(t)$ as the inputs and the angular velocity of the motor shaft $\omega(t)$ as the output. The notations carry their usual meanings. [10]



Q2. Use Pade approximation method to reduce the following stable transfer function to a transfer function having two poles and two zeros. [10]

$$G(s) = \frac{10s + 4}{4s^3 + 6s^2 + 2s + 2}$$

Q3. The open loop dynamics of an LTI system is given by

$$\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1] \underline{x}$$

Design a linear state feedback controller for the system such that the closed loop impulse response decays as quickly as e^{-2t} . Also determine the steady state error of the closed loop system to a unit step reference input. [10]
