BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

Modern Control Systems (EEE F422) [1st Semester, 2022-2023]

Comprehensive Exam: Part A (Closed Book)

Max Time: 1.5 Hr | Max Marks: 50 | Date: 27/12/22

(All notations and abbreviations carry their standard meanings)

Q1. a) Is the matrix identity $[(A+BC)D]^T = D^T[(A^T + B^T C^T)]$ true or false? If false, write the correct one. [1]

b) What do you mean by balanced realization and balanced truncation? [2]

c) Write two disadvantages of linear state feedback control law using pole placement technique. [2]

d) What are the two main limitations of Lyapunov's direct method for stability analysis? [2]

e) Is backstepping a robust control law? Justify. Is it applicable to linear or nonlinear systems? [2]

f) What do you mean by a white noise process? Why is it called white? [2]

g) An LQG regulator is designed for a 4th order system having three controlled inputs, two disturbance inputs, and one output. What will be the dimensions of the Q, R, W, V matrices? [2]

h) What is bifurcation? Distinguish between saddle node and transcritical bifurcations. [2]

Q2. a) Derive the formula for discretization of a continuous time state space representation of an LTI system using the ZOH method. [4]

b) Applying the formula derived in Q2.a), obtain the discrete time model of the following continuous time model for a sampling interval of $0.1s$. [4]

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1]\, \underline{x} + 2u$$

c) Obtain a state space representation of the transfer function $G(s) = \frac{e^{-2s}}{(s+1)(s+2)}$. [4]

Q3. a) Classify the equilibrium point (stable focus, unstable focus etc.) of the system $\underline{\dot{x}} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \underline{x}$. Assume four different initial conditions in four different quadrants and draw rough sketches of the corresponding phase trajectories. [5]

b) Draw the output waveform of relay with deadzone nonlinearity when the input is a sinusoid. Label the waveform properly. (Need not derive the describing function). [3]

c) Apply both indirect and direct (or 1st and 2nd) methods of Lyapunov to determine the stability of the equilibrium point of the following system and comment on your findings. [5]

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - x_1^2\, x_2$$

Q4. a) Consider an LTI system given by the dynamic model [6]

$$\dot{x} = -x + u \; ; x(0) = 1$$

Determine the optimal control signal $u(t)$ which will transfer the state to the origin in $1s$ minimizing the performance index

$$J = \int_0^1 (x^2 + u^2)\, dt$$

b) Write a short note on Extended Kalman Filter. [4]

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Modern Control Systems (EEE F422) [1st Semester, 2022-2023]

Comprehensive Exam: Part-B (Open Book)

Max Time: 1 Hr | Max Marks: 30 | Date: 27/12/22

Q1. For a 3rd order nonlinear system given by [12]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -x_1^2 + x_2 x_3 + u$$
$$y = x_1$$

design a backstepping control such that the output tracks a time profile $y(t) = \sin t$.

Q2. The open loop dynamics of a 2nd order LTI system is given by [8]

$$\dot{\underline{x}} = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = [0 \quad 1]\, \underline{x}$$

Design a state observer for the system choosing the observer poles appropriately.

Q3. Consider the following system whose states are being estimated using a Kalman filter. [10]

$$x_{1,k} = x_{1,k-1} + x_{2,k-1}$$
$$x_{2,k} = -x_{1,k-1} + w_{k-1}$$
$$z_k = x_{1,k} + v_k$$

where w_k and v_k are white noise processes with strengths 20 and 10 respectively. Given the initial state estimate $\underline{\hat{x}}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and initial a-posteriori estimation error covariance $P_0 = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$, compute $\underline{\hat{x}}_1$ and $\underline{\hat{x}}_2$ (i.e. estimates of the states at $k = 1$ and $k = 2$) provided z_1 = -0.5 and z_2 =0.5.

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