BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

Modern Control Systems (EEE F422) [1st Semester, 2023-2024]

Mid-Semester Test: Part-A (Closed Book)

Max Time: 45 min | Max Marks: 30 | Date: 10/10/23

(Answer all questions. All notations and abbreviations carry their standard meanings.)

Q1. (i) State one major advantage of state space representation over transfer function representation.

(ii) The following state space model of an LTI system is in CCF/OCF/JCF/None?

$$\underline{\dot{x}} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \, ; \; y = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{x}$$

(iii) What do you mean by equilibrium point of a dynamic system?

(iv) What do you mean by pitchfork bifurcation?

(v) For a 2nd order system, a Lyapunov function is chosen as $V = x_1^2 + x_2^2 + 1$. Is it a valid Lyapunov function? Give justification. [5×1=5]

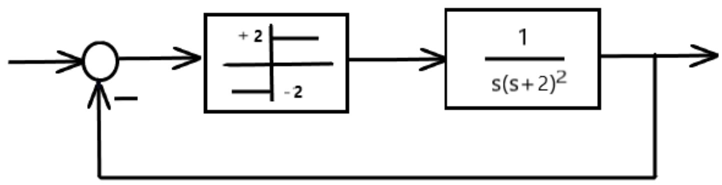
Q2. (i) Solve the following differential equation using state space method:

$2\ddot{y}(t) + 6\dot{y}(t) + 2y(t) = u(t)$ with $y(0) = 1$, $\dot{y}(0) = -1$ where $u(t)$ is a unit step function.

(ii) Derive the formulae for conversion of a continuous-time state space model of an LTI system to a discrete-time state space model retaining the step response of the original continuous-time system. [8+4=12]

Q3. (i) Derive the describing function of an ideal relay (shown in the Fig.).

(ii) Using the results of (i), compute the amplitude and frequency of the limit cycle in the closed loop system as shown in the figure below. Is the limit cycle stable?



(iii) Apply Lyapunov's direct method to determine a region in the state space where the equilibrium point of the system $\dot{x} = (1 - x)^3 + x$ is asymptotically stable. [4+6+3=13]

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Modern Control Systems (EEE F422) [1st Semester, 2023-2024]

Mid-Semester Test: Part-B (Open Book)

Max Time: 30 Min | Max Marks: 20 | Date: 10/10/23

(Answer all questions. All notations and abbreviations carry their standard meanings.)

Q1. An armature controlled separately excited DC Motor is coupled with a simple pendulum as shown in the figure. The motor provides the torque (T_e) to rotate the pendulum in the vertical plane where the angular position of the pendulum can vary in the range [-60^0, $+120^0$]. The mechanism represents a single-link robotic manipulator.

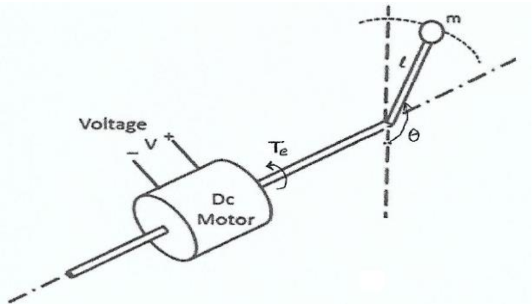
(i) Derive a state space model of the system considering armature voltage as the input and angular position of the bob as the output assuming the following parameters:

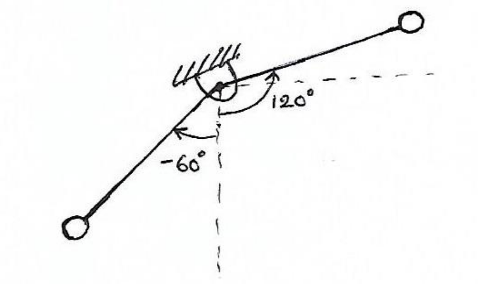
Armature resistance, R_a=1Ω
Armature inductance, L_a=1H
Torque constant and back emf constant of the motor, $k_t = k_b$ =1 unit
Inertia of the motor shaft, J= 1 kg-m^2
Coefficient of viscous friction in the motor shaft, B= 1 N-s/m (pendulum hinge is frictionless)
Acceleration due to gravity, g=10 m/s^2
Length of the pendulum rod, l=1m (the rod to be massless)
Mas of the pendulum bob, m=1 kg

(ii) Determine how much voltage should be fed to the motor to hold the pendulum at

(a) $\theta = -60^0$
(b) $\theta = 120^0$

(iii) Now when the pendulum is held at the position $\theta = -60^0$, if the bob is slightly perturbed, will it come back to $\theta = -60^0$? Similarly, when the pendulum is held at $\theta = -120^0$, if the bob is slightly perturbed, will it come back to $\theta = 120^0$? [5+4+5=14]





Q2. For the following LTI system, find the conditions on k_1, k_2, k_3, k_4 for the system to be observable. When the conditions are violated and the system loses observability, will it be detectable? [6]

$$\underline{\dot{x}} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \; ; \quad y = [k_1 \quad k_2 \quad k_3 \quad k_4]\, \underline{x}$$

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