BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

Modern Control Systems (EEE F422) [1st Semester, 2023-2024]

Comprehensive Exam: Part-A (Closed Book)

Estimated Time: 2 Hrs   Max Marks: 60   Date: 08/12/23

Answer all questions. All notations and abbreviations carry their standard meanings.
All parts of Q1 must be answered at one place.

Q1. (i) An 8th order LTI system has poles at $-0.5 \pm j0.5$, $-0.5 \pm j0.5$, $-2, -2, -2, -5$. How will the system matrix A look like if the system is represented in Jordan canonical form? [3]

(ii) Determine the sign definiteness of the quadratic function $x_1^2 + x_2^2 - x_1 x_2$. [1]

(iii) What do you mean by stabilizability and detectability? [2]

(iv) Write the dimensions of the controllability and observability matrices for a two input single output 3rd order LTI system. [2]

(v) For the state space model $\dot{\underline{x}} = A\underline{x} + Bu$; $y = C\underline{x} + Du$ of an LTI SISO system, derive the formula for computing the transfer function of the system. Name the MATLAB command available for this purpose. [3]

(vi) What is the full form of LQG regulator? [1]

(vii) Which of the properties of an LTI system among controllability, observability and stability get affected by similarity transformation? [1]

(viii) What are limit cycles? [2]

(ix) What is the utility of LaSalle's invariance principle for stability analysis of nonlinear systems? [1]

(x) What is describing function? What is its usefulness? [2]

(xi) Step response is preserved in the conversion of Continuous Time (CT) model to Discrete Time (DT) model using ZOH method. True/False? What is the MATLAB command available to convert CT models to DT models? [2]

(xii) Compute the first four time moments of the transfer function $\frac{1}{s^2+1}$. [2]

Q2. Convert the state space model of a 2nd order LTI system as given below to DCF and therefrom obtain an approximate 1st order model using residualization method so that the DC gain is preserved.

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -10 & -11 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1]\, \underline{x}$$

[12]

Q3. A 2nd order nonlinear dynamic system is described by the following state equations

$$\dot{x}_1 = -2\, sin\, x_1 + x_1^3 + x_2$$

$$\dot{x}_2 = -x_2^2 - x_1^2 x_2 + u$$

where x_1 always remains within the range $\pm 1\, rad$. The objective is to keep x_1 at zero. Assuming both the states are measurable, design a backstepping control such that the favorable terms in the dynamics are not cancelled and only the unfavorable terms are cancelled. [14]

Q4. (i) As we know, a state estimator accepts the plant input and plant output as input signals and generates the estimate of the state as the output signal. Therefore, if the estimator is visualized as a dynamic system in itself, obtain the general expression of its transfer function. Consider a linear scenario. [4]

(ii) Consider a dynamic system described by

$$\dot{x} = 2x + u - 2\dot{u} \; ; \; x(0) = 2$$

Determine the optimal control $u^*(t)$ which will transfer the initial state to the origin in $2 \, sec$ minimizing the total control energy $J = \frac{1}{2}\int_0^2 u^2 \, dt$ [8]

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Modern Control Systems (EEE F422) [1st Semester, 2023-2024]

Comprehensive Exam: Part-B (Open Book)

Estimated Time: 1 Hr | Max Marks: 30 | Date: 08/12/23

(Answer all questions. All notations and abbreviations carry their standard meanings.)

Q1. (i) A general nonlinear MIMO dynamic system is described by

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u})$$
$$\underline{y} = \underline{h}(\underline{x}, \underline{u})$$

Develop a theory to design a local linear state observer. Clearly explain how the obsererver gain L can be designed and also derive the observer equation. [8]

(ii) Apply the theory developed in Q1(i) above, to design a linear state observer for the following system:

$$\dot{x}_1 = -x_1 + e^{-u}x_2$$

$$\dot{x}_2 = x_1x_2 + u$$

$$y = x_1 + x_2 + (1 + x_1x_2)u$$

Design the observer L near the equilibrium point $x_1 = 0, x_2 = 0, u = 0$ ensuring that the estimation errors decay at least at a rate of e^{-2t}. Also write the observer equation. [8]

Q2. (i) Consider a 1st order LTI system given by

$$\dot{x} = x + u$$

The objective is to design an LQR such that the performance index, $J = \frac{1}{2}\int_0^\infty (qx^2 + ru^2)\, dt$ is minimized.

Show that the resulting controller gain will be high for high values of q and low values of r. [5]

(ii) Consider a discrete time stochastic system given by

$$\underline{x}_k = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \underline{x}_{k-1} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{k-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_{k-1}$$

$$\underline{z}_k = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x}_{k-1} + v_k$$

where the strengths of the white process disturbance and white measurement noise are 10 and 5 respectively. Assuming the initial estimation error covariance to be $P_0 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, compute the Kalman gain, a-priori estimation error covariance and a-posteriori estimation error covariance for $k = 1,2$. [9]

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