Comprehensive Exam (Part−II)

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Course No./Title : EEE F430/Green communications and Networks
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Note: Answer the following questions. Provide key intermediate steps. ($5 \times 5 = 25$ points)

Q. 1. Figure 1 depicts the wireless power transfer (WPT) circuit based on inductive coupling. Recall the Tesla strategy. Suppose that for an incremental subinterval $\Delta\omega$, $|I_\ell(\Delta\omega)| \approx |I_s(\Delta\omega)|$. Let $L_2 = \beta M, 0 < \beta < 1$, $r_\ell C_\ell = \tau_\ell$. *Answer the following:*
(i) Obtain a quadratic equation in M in the form $\mathcal{A}M^2 + \mathcal{B}M + \mathcal{C} = 0$. Explicitly write the real factors $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, which depend on the WPT circuit parameters. Determine the roots.
(ii) Determine the conditions which guarantee that out of the two real roots, one is negative and one is positive.

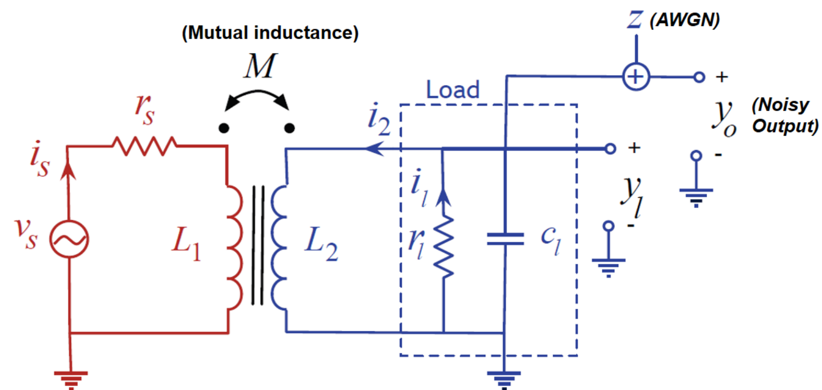

Fig. 1: Pertaining to Q.1.

(iii) Assume that, for the measured subinterval $\Delta\omega$, $|I_\ell(\Delta\omega)|^2 \approx |I_s(\Delta\omega)|^2$. Obtain an approximate expression for the ratio of r_ℓ and r_s in terms of $\eta(\Delta\omega)$. For $\eta(\Delta\omega) = 0.9$, determine r_ℓ when $r_s = 1\ k\Omega$.

[3 + 1 + 1 points]

Q. 2. Consider the fast fading wireless channel scenario. The optimization problem is defined as follows.

$$\max_{P_1, P_2, \ldots, P_L \geq 0} \frac{1}{L} \sum_{\ell=1}^{L} \log_2 \left(1 + \frac{P_\ell \gamma_\ell}{\sigma_n^2 + \sigma_i^2}\right),$$

$$\text{such that } \frac{1}{L} \sum_{\ell=1}^{L} P_\ell = \overline{P}, \text{ for all } \gamma_\ell.$$

Use the following notation.

- L denotes the number of parallel subchannels that fade independently.
- $P_\ell, \ell = 1 \leq \ell \leq L$ denotes the power allocation for ℓ^{th} subchannel.
- $\gamma_\ell \geq 0$ denotes ℓ^{th} subchannel power gain.
- σ_n^2 denotes the white Gaussian noise variance and σ_i^2 denotes the interference variance.
- $\overline{P}$ is the average power constraint.

Answer the following: (i) Let P_ℓ^* is the optimal power allocation for ℓ^{th} subchannel. Using the Lagrangian approach, obtain the water−filling solution in terms of $\lambda^* > 0$, γ_ℓ, for $\sigma_n^2 = 0$ dB and $\sigma_i^2 = 0$ dB. How can you determine λ^*? Comment. Write down the average power constraint using P_ℓ^* as $L \to \infty$.
(ii) Suppose the transmitter has knowledge of CSI (i.e.γ) and transmits with adaptive power P_t. Write down the expressions for fading−averaged spectral efficiency (FASE) and fading−averaged energy efficiency (FAEE). *Note:* Assume that P_{est} denotes the power consumed for acquiring CSI. Further, assume that P_{crt} denotes the power consumed by the circuitry.

[3 + 2 points]

Q. 3. Consider a dual hop, TS−SWIPT cooperative wireless system as shown in the Fig. 2. In it, all are single antenna nodes, and the SWIPT relay is half−duplex. The source transmits at a fixed power P_s. Without loss of generality, let the noise variance at the relay node is unity. Let the frequency−flat fading channel power gain $\gamma_{sr} \triangleq |h|^2 \sim \exp(1)$ and the first hop path loss factor is denoted by $L_p^{\mathbb{S}\to\mathbb{R}}$. Let Γ_R denote the instantaneous SNR at the EH relay receiver, and the conversion efficiency is η.

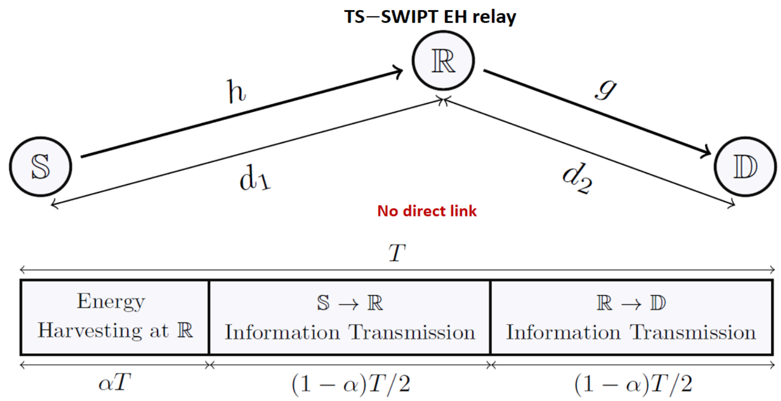

Fig. 2: Pertaining to Q.3.

Determine the following: (i) $\mathbf{E}\left[\Gamma_R^2\right]$, (ii) $\mathbf{var}\left[\Gamma_R\right]$ and standard deviation, (iii) $\frac{\mathbf{var}[\Gamma_R]}{(\mathbf{E}[\Gamma_R])^2}$. Comment on the simplified result.

[2 + 2 + 1 points]

Q. 4. Consider packet data transmission over AWGN channel with bit rate $R_b \approx \mathcal{C}$, channel capacity. Denote the average bit error rate of $M-$QAM modulation as p_{be}. Assume that bit errors are statistically independent and uniformly distributed. Let Γ denote the link SNR at the destination. *Answer the following:*

(i) Let the packet has L bits. Obtain an approximate expression for the packet error rate (p_{pe}) in terms of M, Γ and L.

(ii) Let ρ_{oh} denote the percentage of overhead bits. Note that each packet contains L bits, including data bits and overhead bits. Obtain the approximate expression for SE in terms of p_{pe}, ρ_{oh} and SNR (Γ).

Note: The end−to−end power consumption $= P_t + P_{crt}$, where $P_{crt} \approx 5\%$ of P_t. Obtain the expression for EE.

(iii) *Consider:* SNR $\Gamma = 10$ dB, $4-$QAM, $\rho_{oh} = 10\%$, $L = 90$ bits, Total power consumption $= 10$ milliwatts. Compute EE.

[2 + 2 + 1 points]

Q. 5. Refer to the cooperative and cognitive system model shown. Assume statistically independent frequency−flat fading channels and the coherent maximal ratio combining (MRC) receiver. All nodes have a single antenna, and the relay is half−duplex. Let P_s be the source transmit power. Let $\gamma_{sd} \triangleq |h_{sd}|^2$, $\gamma_{sr} \triangleq |h_{sr}|^2$, and $\gamma_{rd} \triangleq |h_{rd}|^2$. Further, Γ_{sd}, Γ_{sr} and Γ_{rd} denote the SNRs of the $S-D$, $S-R$ and $R-D$ links, respectively. Assume Gaussian noise and interference model. *Answer the following:*

(i) Write down the expression for Θ. Assume the fixed−power relay model where $\overline{P}_r$ is the average relay transmit power.

(ii) Let $\delta_1 = \sigma^2 + \sigma_r^2$ and $\delta_2 = \sigma^2 + \sigma_d^2$. Obtain an expression for SNR Γ_E at the destination in terms of $\Gamma_{sd} \triangleq P_s\gamma_{sd}$, $\Gamma_{sr} \triangleq P_s\gamma_{sr}$, and $\Gamma_{rd} \triangleq \overline{P}_r\gamma_{rd}$ and noise and interference variances. Write down the expressions for FASE and FAEE. Use P_T to denote the total power.

(iii) Consider SWIPT TS−based EH relay with $\alpha = 0.6$. Additional parameters include: $P_s = 10$ dB, $L_p^{\mathbb{S}\to\mathbb{R}} = 0.18$, $\gamma_{sr} \sim \exp(1)$, $\eta = 54$ %. The estimated channel power gains are $\hat{\gamma}_{sd} \approx 0.1$, $\hat{\gamma}_{sr} \approx 1$, $\hat{\gamma}_{rd} \approx 1$. Further, assume noise and interference variances are equal to unity. Compute: $\overline{P}_r$ (in dB), Γ_E and estimated EE. *Note:* Assume that the entire harvested energy is used for transmission. Total power consumption $= 30$ dBm.

[2 + 2 + 1 points]

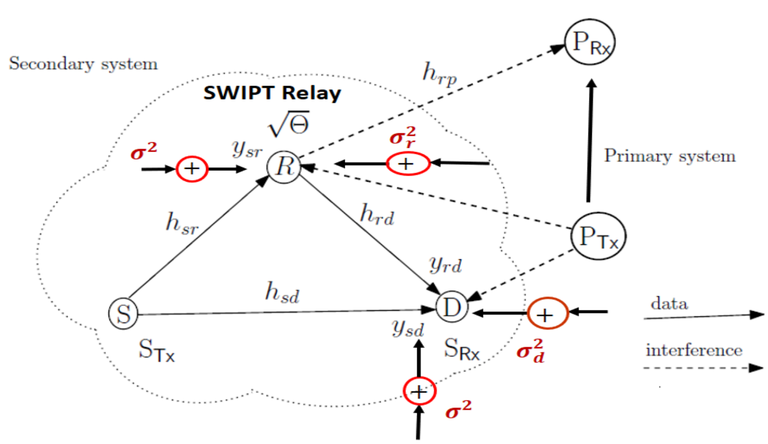

Fig. 3: Pertaining to Q.5.

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