Birla Institute of Technology and Science Pilani (Rajasthan)

EEE F431 Mobile Telecom Network
Second Semester 2021-22
Date : 15/03/2022 **Mid Semester Exam** **Max. Marks: 50**

Name: ________________________________ **ID No.** ________________ **Time: 90 min**

Instructions:

1) This is an open book exam. Use of textbook (Wireless Communication Principles and Practice, Rappaport), reference book (Wireless communication, Goldsmith), and hand written notes is allowed.

2) Show all the steps clearly. If I cannot interpret it, I cannot grade it.

Q.1a) Consider a Rayleigh fading channel where the PDF of signal envelope A is given by $f_A(a) = 2ae^{-a^2}$, $0 \leq a \leq \infty$. Derive the PDF of power gain given by $g = a^2$. **(3)**

Q.1b) For the Rayleigh channel in above question, consider a transmit power $P_{\mathrm{t}} = 20$ dB. What is the probability that the power at the receiver is greater than $P_r = 10$ dB ? **(3)**

Q.1c) Consider an OFDM system with total passband bandwidth $B = 5$ MHz having $N = 512$ subcarriers. The channel has a maximum delay spread of $T_m = 4\,\mu s$. Answer the following questions.

I. What is the raw symbol time without the cyclic prefix, and what are the minimum number of samples required in the cyclic prefix? **(1)**

II. If the length of the cyclic prefix is twice the required minimum calculated above, what is the total OFDM symbol time? **(1)**

III. What is the reduction in data rate due to the overhead of the cyclic prefix? **(1)**

IV. If the modulation employed is 16-QAM, what is the effective bit rate of the above OFDM system? **(2)**

Q.2a) Consider the MIMO channel wherein the singular value matrix obtained from the SVD of **H** matrix is given as,

$$\sum = \begin{matrix} \sqrt{52} & 0 & 0 \\ 0 & \sqrt{13} & 0 \\ 0 & 0 & 2 \end{matrix}$$

Considering a transmit power of $\mathrm{P} = -1.25$ dB and noise power $\sigma_n^2 = 3$ dB, compute the MIMO capacity and optimal power allocation. **(5)**

Q.2b) Assume that, at a certain distance, we have a deterministic propagation loss of 127 dB and large-scale fading, which is log-normally distributed with $\sigma_{\psi_{dB}} = 7$dB.

I. How large is the outage probability (due to large-scale fading) at that particular distance, if our system is designed to handle a maximal propagation loss of 135 dB? **(3)**

II. Which of the following alternatives can be used to lower the outage probability of our system, and why are they/are they not possible to use? **(4)**

IIa. Increase the transmit power.
IIb. Decrease the deterministic path loss.
IIc. Change the antennas.
IId. Lower the $\sigma_{\psi_{dB}}$

Q.3a) The average BER for BPSK in maximum ratio combining can be approximated as,

$$\overline{BER} \approx \left(\frac{1}{4\bar{\gamma}}\right)^{N_r} (2\,N_r - 1)\,C_{N_r}$$

Assume that the average SNR $\bar{\gamma}$ is 20 dB.

I. Compare the average BER for $N_{\mathrm{r}} = 1$ and $N_{\mathrm{r}} = 3$ receive antennas. Which of these is a better setting? **(2)**

II. Calculate the SNR that would be required in a one-antenna system in order to achieve the same BER as a three-antenna system at 20 dB. **(2)**

Q.3b) Consider a switched diversity system where the receiver with 2 antennas takes as its input the signal from one antenna as long as the SNR is above some threshold. When the SNR falls below the threshold, the receiver switches to the second antenna, regardless of the SNR at the second antenna. If the antennas fade independently and according to the same distribution, it can be shown that the CDF of the SNR at the RX is,

$$F_\gamma(\gamma) = \begin{cases} \Pr(\gamma_1 \le \gamma_t \; and \; \gamma_2 \le \gamma), & for \; \gamma < \gamma_t \\ \Pr(\gamma_t \le \gamma_1 \le \gamma \; or \; [\gamma_1 \le \gamma_t \; and \; \gamma_2 \le \gamma]), & for \; \gamma \ge \gamma_t \end{cases}$$

where γ is the SNR after the switching device (i.e., the SNR at the receiver), γ_1 is the SNR of the first antenna, γ_2 is the SNR of the second antenna, and γ_t is the switching threshold.

I. For Rayleigh fading where both antennas have the same mean SNR $\bar{\gamma}$, what is the CDF and PDF of γ. **(2)**

II. What is the optimum switching threshold that maximizes the mean SNR and the resulting mean SNR? What is the gain in dB of using switched diversity compared with that of a single antenna? Compare this gain with that of maximal ratio combining and selection diversity. **(4)**

Q.4a) Consider the wireless system given as,

$$y = [\,1 + j \quad 3 + 4j\,]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n$$

Indicate the step by step processing at the transmitter and receiver for above system using Alamouti codes and show how the input signal can be recovered. **(5)**

Q.4b) The power delay profile of an urban channel impulse response consists of two clusters, each having exponential decay on a linear scale as,

$$P(\tau) = \begin{cases} a_1 e^{-b_1 \tau} & 0 \le \tau \le 20\ \mu s \\ a_2 e^{-b_2[\tau - 55\times 10^{-6}]} & 55\ \mu s \le \tau \le 65\ \mu s \\ 0 & else \end{cases}$$

Given that $P(0), P(20\mu s), P(55\ \mu s)$, and $P(65\mu s)$ are respectively, $10^{-13}W, 10^{-17}W, 10^{-15}$W, and $10^{-17}W$. Answer the following.

I. Find coefficients $a_i s$ and b_is. **(2)**

II. Calculate the time integrated power $\int_{-\infty}^{\infty} P(\tau) d\tau$. **(1)**

III. Calculate the mean delay and RMS delay spread **(4)**

IV. Would you characterize the channel as frequency flat or frequency selective for a system bandwidth of 100 kHz. Justify your answer. **(1)**

Q.5) A radio system is usually specified in such a way that a receiver should be able to handle a certain amount of Doppler spread in the received signal, without losing too much in performance. Assume that only the mobile receiver is moving and that the maximal Doppler spread is measured as twice the maximal Doppler shift. Further, assume that you are designing a mobile communication system that should be able to operate at both 900 MHz and 1800 MHz.

I. If you aim at making the system capable of communicating when the terminal is moving at 200 km/h, which maximal Doppler spread should it be able to handle? **(2)**

II. If you design the system to be able to operate at 200 km/h when using the 900 MHz band, at what maximal speed can you communicate if the 1800 MHz band is used (assuming the same Doppler spread is the limitation)? **(2)**

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