BITS, Pilani – K. K. Birla Goa Campus
EEE F435 DIGITAL IMAGE PROCESSING
Semester I 2022-23, Comprehensive Exam
Date: December 30, 2022; Time: 10:00 am – 1:00 pm

1. **(i)** Consider an image $f(x,y)$ of size $M \times N$. **Implement** the gradient $\boldsymbol{\nabla_{x,y}}$ and the Laplacian $\boldsymbol{\nabla^2_{x,y}}$ of $f(x,y)$ by spatial filtering using two 1-D spatial masks respectively. Write the **expressions** and **coefficients** of the two 1-D spatial masks $\{\boldsymbol{h_{\nabla_x}(x)}$ and $\boldsymbol{h_{\nabla_y}(y)}\}$ for $\nabla_{x,y}$ and the two 1-D spatial masks $\{\boldsymbol{h_{\nabla^2_x}(x)}$ and $\boldsymbol{h_{\nabla^2_y}(y)}\}$ for $\nabla^2_{x,y}$. Find the **expressions** for the equivalent 1-D frequency domain filters: $\boldsymbol{H_{\nabla_x}(u)}$, $\boldsymbol{H_{\nabla_y}(v)}$, $\boldsymbol{H_{\nabla^2_x}(u)}$ and $\boldsymbol{H_{\nabla^2_y}(v)}$. ------- **(6 Marks)**

   (ii) Consider the binary image in Fig. 1-a, $f(x,y)$ of size 12×32 consisting of alternating stripes of white (1) and black (0) in both spatial directions, each stripe being 2 pixels wide. Find the frequency domain coordinates $(\boldsymbol{u},\boldsymbol{v})$, where $u = -6,-5,\ldots,-1,0,1,\ldots,5$ and $v = -16,-15,\ldots,-1,0,1,\ldots,14,15$, of the **three most dominant frequency components in the centered-DFT of** $\boldsymbol{f(x,y)} \overset{\mathcal{F}}{\Leftrightarrow} \boldsymbol{F(u,v)}$. Write the **magnitude of the highest frequency component in** $\boldsymbol{F(u,v)}$. Find the **frequency domain mask** $\boldsymbol{H(u,v)}$ required to filter out the white stripes in the horizontal direction from $f(x,y)$ to obtain the output image $g(x,y)$ shown in Fig. 1-b. Write the **steps involved in filtering**. ------------- **(14 Marks)**

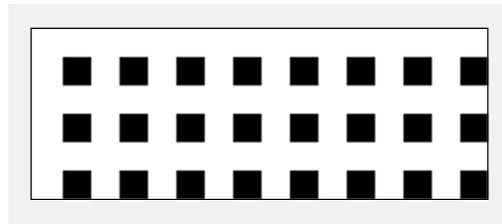


Fig. 1-a

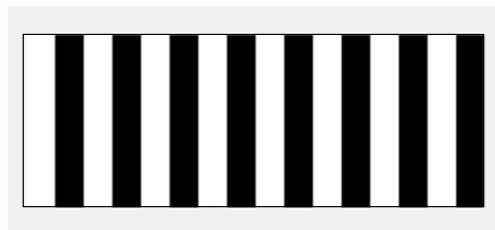


Fig. 1-b

   Note: The black borders are not part of the images. For $H(u,v)$, it is sufficient to write the locations where $H(u,v)$ is zero and non-zero and the magnitudes of the non-zero values.

2. Consider the 4×6, 8-bit image $f(x,y)$ given below. A 512-word dictionary with the starting content shown in Fig. 2-a is assumed. Locations 256 through 511 initially are unused. Compress the image using the LZW coding and **complete the table shown in Fig. 2-b**. Calculate the compression ($\boldsymbol{C_{LZW}}$) achieved. ------- **(15 Marks)**

$$f(x,y) = \begin{bmatrix} 21 & 21 & 95 & 169 & 243 & 243 \\ 21 & 21 & 95 & 169 & 243 & 243 \\ 21 & 21 & 95 & 169 & 243 & 243 \\ 21 & 21 & 95 & 169 & 243 & 234 \end{bmatrix}$$

   Compress the encoded output (i.e., the third column in the table in Fig. 2-b) using **quaternary {0, 1, 2, 3} Huffman coding**. Show all the steps involved in the Huffman coding. Compute the compression ($\boldsymbol{C_H}$) achieved. ----- **(5 Marks)**

Dictionary Location	Entry
0	0
1	1
⋮	⋮
255	255
256	—
⋮	⋮
511	—

Fig. 2-a

Currently Recognized Sequence	Pixel Being Processed	Encoded Output	Dictionary Location (Code Word)	Dictionary Entry

Fig. 2-b

3. **(i)** Consider the image I with an object A and the eight structuring elements B_i for $i = 1,2,\ldots,8$, their origins are indicated by the underlined pixels. Compute the morphological operations: ----------- **(5 Marks)**

a) $A_1 = \{z|(B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c\}$

b) $A_2 = (A \ominus B_3) \cap (A^c \ominus B_4)$

c) $A_3 = (A \ominus B_5) - (A^c \ominus B_6)^c$

d) $A_4 = (A \ominus B_7) - (A^c \oplus \hat{B}_8)$

e) $A_5 = A_1 \cup A_2 \cup A_3 \cup A_4$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	1	1	1	1	0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	0	0	1	1	1	1	1	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0	1	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0	1	1	1	1	0	0	0
0	0	0	1	0	0	0	0	0	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

I

×	1	×
0	<u>**1**</u>	1
0	0	×

B_1

×	0	×
1	<u>**0**</u>	0
1	1	×

B_2

×	1	×
1	<u>**1**</u>	0
×	0	0

B_3

×	0	×
0	<u>**0**</u>	1
×	1	1

B_4

×	0	0
1	<u>**1**</u>	0
×	1	×

B_5

×	1	1
0	<u>**0**</u>	1
×	0	×

B_6

0	0	×
0	<u>**1**</u>	1
×	1	×

B_7

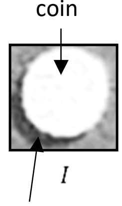
1	1	×
1	<u>**0**</u>	0
×	0	×

B_8

(ii) Consider the 4-bit image $I(x,y)$ given below containing a coin and its shadow in a background (without the dark border). Using image segmentation techniques (which includes point, line and edge detection, and thresholding), **segment the image into two binary images: I_1, consisting of only the coin in a background and I_2, consisting of only the shadow in a background**. **Show all the steps involved and write the two output images**. ------- **(15 Marks)**

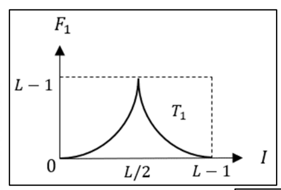
$$I(x,y) = \begin{bmatrix} 8 & 7 & 7 & 8 & 14 & 14 & 14 & 8 & 8 & 8 \\ 8 & 8 & 7 & 14 & 15 & 12 & 13 & 14 & 9 & 8 \\ 7 & 4 & 13 & 12 & 14 & 14 & 14 & 15 & 14 & 8 \\ 7 & 3 & 14 & 13 & 13 & 13 & 14 & 14 & 13 & 8 \\ 7 & 3 & 12 & 13 & 13 & 12 & 13 & 14 & 12 & 8 \\ 7 & 4 & 4 & 13 & 12 & 13 & 12 & 14 & 8 & 7 \\ 7 & 6 & 4 & 4 & 12 & 13 & 14 & 7 & 8 & 8 \\ 6 & 7 & 6 & 4 & 4 & 4 & 3 & 7 & 7 & 6 \\ 6 & 7 & 7 & 7 & 6 & 6 & 7 & 7 & 7 & 6 \end{bmatrix};$$

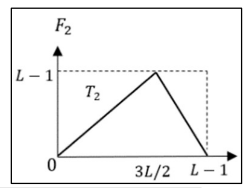
coin

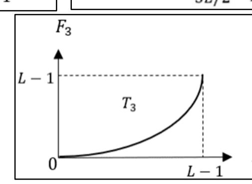


I

shadow







If the image I is corrupted by a Gaussian noise with mean 7 and a standard deviation of 4 intensity levels, **would your segmentation still work? Explain**.

The intensity transformations T_i for $i = 1, 2, 3$ are used on the input image I to obtain the output images F_i. **Would your segmentation still work on the output images F_i i.e., after the transformations? Explain**.

4. **(i)** Find the **shape number** and **order of the shape number** for the given boundary (Use 4-directional code) in Fig. 4-a. Each line segment is of equal length. -------- **(4 Marks)**

(ii) Consider the feature matrix x representing a set of four observations (samples) of three features of an object. Each row in x represents a sample.

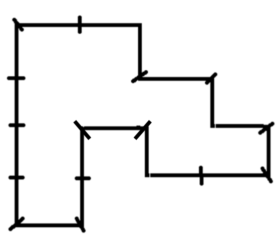


Fig 4-a

$$x = \begin{bmatrix} 8 & 6 & 1 \\ 8 & 5 & 7 \\ 7 & 2 & 9 \\ 8 & 2 & 2 \end{bmatrix}$$

a) Calculate the **covariance matrix for x**. -------- **(4 Marks)**

b) Find the transformation from x to y using the Hotelling Transform relation. Write the **transformation matrix A**. -------- **(8 Marks)**

c) Calculate the **covariance matrix for y**. ---------- **(4 Marks)**