# COMPREHENSIVE EXAMINATION

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Course No./Title : EEE F472/Satellite communications
DATE: Dec. 8$^{\text{th}}$, 2023. Test duration: 180 Mins. Max. Points: 40.

(*Note*: You may use standard results or formulae by stating them explicitly. Highlight final answers in boxes. Simplify your response to the extent possible. Use notation consistently. Provide key intermediate steps.)

**Q. I. [5 + 4 points]** Consider the non−linear characteristic of the transponder high power amplifier. The following relationship holds between the input $v_i$ and the output $v_0$: $v_o = v_i + v_i^2$. Three unmodulated, unit-amplitude cosine carriers at frequencies $f_1$, $f_2$, and $f_3$ ($f_3 > f_2 > f_1$) are applied at the input of the power amplifier. *Answer the following:*

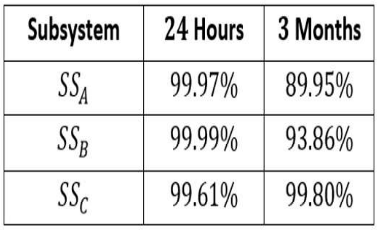
**Part−(a)**. i) Determine the output signal expression (in the most simplified form that includes DC and AC terms.). *Note:* Do not show any square terms in the final expression.

ii). How many intermodulation (IM) terms are present? How many associated spectral components are present? Write down the IM terms. *Note:* Mention only the IM terms.

**Part−(b)**. i) Suppose $f_1 = 3700$ MHz, $f_2 = 3710$ MHz, $f_3 = 3720$ MHz. Present the IM frequencies in the form of a table. *Note:* Make a table with the three columns: S.No., IM term, IM frequency.

ii) Determine the ratio of the power amplifier input's total average (unmodulated) carrier power to the total average power of the IM frequency terms (in dB).

**Q. II. [3 + 6 points]** Consider the design of a complex spacecraft system comprising of three subsystems labeled as $\mathcal{SS}_{\mathcal{A}}$, $\mathcal{SS}_{\mathcal{B}}$, $\mathcal{SS}_{\mathcal{C}}$. The following table provides the probability of success of each subsystem after time periods of operation. *Answer the following:*



| Subsystem | 24 Hours | 3 Months |
|---|---|---|
| $SS_A$ | 99.97% | 89.95% |
| $SS_B$ | 99.99% | 93.86% |
| $SS_C$ | 99.61% | 99.80% |

**Tab. Probability of success for subsystems**

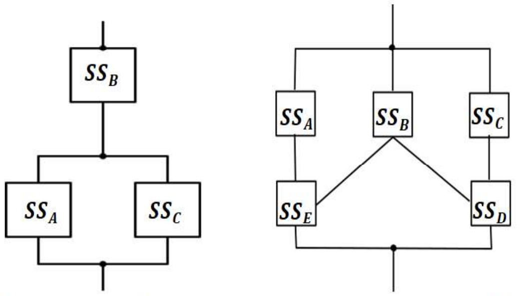


**Fig. System for Part−($a$)** **Fig. System for Part−($b$)**

Fig. 1: Pertaining to Q. II.

**Part−(a)**. i) Refer to the first system depicted in the figure (a). It has subsystems $\mathcal{SS}_{\mathcal{A}}$, $\mathcal{SS}_{\mathcal{B}}$, and $\mathcal{SS}_{\mathcal{C}}$. Compute the probability of success ($p_s$) for this system for the three months.

ii). Consider the reliability function (in months) defined as $\mathcal{R}(t) = \exp\left(-\left(1-p_s\right)t\right)$, where $p_s$ is determined in (i). Determine the mean time before failure (MTBF in months).

**Part−(b)**. Refer to the extended system having five subsystems as depicted in figure (b). The probability of success for $\mathcal{SS}_{\mathcal{A}}$, $\mathcal{SS}_{\mathcal{B}}$, and $\mathcal{SS}_{\mathcal{C}}$ are as in the part-(a). Further, the probability of success (for the three month period) of $\mathcal{SS}_{\mathcal{D}}$, and $\mathcal{SS}_{\mathcal{E}}$, are 95.42% and 92.16%, respectively. *Answer the following:*

i). Determine the conditional probabilities $\mathcal{P}(\text{system succeeds}|\mathcal{SS}_{\mathcal{B}} \text{ succeeds})$ and $\mathcal{P}(\text{system succeeds}|\mathcal{SS}_{\mathcal{B}} \text{ fails})$.

ii). Using the conditional probabilities obtained in (i), compute the probability of success of the system.

iii). Consider the reliability function (in months) defined as $\mathcal{R}(t) = \exp\left(-\left(1-p_s\right)t\right)$, where $p_s$ is determined in (ii). Determine the MTBF in years.

(Please Turn Over)

**Q. III. [5 + 4 points]** Consider the pure ALOHA based variable length packet transmission via digital satellite communication link. Let $\mathcal{H} \triangleq \frac{1}{\mathcal{L}}$ denote the reciprocal of the total offered load to the channel.

Let $\mathcal{U}$ denote the throughput. Assuming highly reliable forward and reverse links, the throughput is given by $\mathcal{U} = \mathcal{H}^{-p}\,(1+\mathcal{H})^{-q}$, where $p = -2$ and $q = 3$. *Answer the following:*

**Part−(a)**. Express $\mathcal{U}(\mathcal{L})$ as a rational fraction. Compute $\mathcal{U}_{\max}$. Let $\mathcal{L}^*$ denote the value of $\mathcal{L}$ at which maximum throughput is achievable. Determine $\mathcal{L}^*$ and $\mathcal{H}^*$. Sketch $\mathcal{U}(\mathcal{L})$ (i.e., throughput as a function of $\mathcal{L}, 0 < \mathcal{L} < 1$.)

ii). Suppose the sender transmission bit rate is 96 Kbps. Determine the effective bit rate at the receiver.

**Part−(b)**. i). Compare the $\mathcal{U}_{\max}$ obtained in (i) of Part−(a) with pure ALOHA (fixed packet transmission). Which one has higher throughout? How much percentage? Justify qualitatively with proper reasoning. Neatly sketch both the throughput curves in a single plot. Indicate $\mathcal{L}^*$ on the plot $x$−axis.

ii). Assume highly reliable satellite packet communication links. Suppose the marginal success probability ($p_{sm}$) of a transmitted packet is given by $p_{sm} = (\mathcal{L}+1)^{-2}$. Compute $p_{sm}$ at $\mathcal{L} = \mathcal{L}^*$.

**Q. IV. [3 + 6 points]** Considered an unshadowed MSAT signal path. Suppose the operating MSAT path has the ratio of direct component power to diffuse component power (that is, the fading factor $\kappa$) equal to 5 dB. *Answer the following:*

**Part−(a)**. i). What is the fading model relevant to the scenario? Why?

ii). Suppose the power in the direct component ($P_{dc}$) is 3 dBW. Let $P_{Dfc}$ denote the power in the diffuse component. What is the total power (in dBW)?

**Part−(b)**. i). Let $\mathcal{Z}$ denote the random variable that models the signal amplitude. The probability density function of the continuous random variable $Z$ is given by

$$p_{\mathcal{Z}}(z) = 2\,(\kappa+1)\,z\exp\{-\left(\kappa + \left((\kappa+1)\,z^2\right)\right)\}\,\mathcal{I}_0\left(2z\sqrt{\kappa\,(\kappa+1)}\right), \quad z \geq 0,$$

where $\mathcal{I}_0(.)$ is the modified Bessel function of zeroth order.

*Notes:* $\int_0^\infty p_{\mathcal{Z}}(z)\,dz = 1$. The following integral is useful:

$$\mathcal{Q}_1(\alpha,\beta) = \int_\beta^\infty y\exp\left\{-\left(\frac{\alpha^2+y^2}{2}\right)\right\}\mathcal{I}_0\left(\alpha y\right)\,dy,$$

where $\mathcal{Q}_{\mathcal{M}}(\alpha,\beta)$ denote the Marcum $Q$−function of order $\mathcal{M}$.

By determining $\alpha$ and $\beta$, compute the percentage of time (in terms of $\mathcal{Q}_1(\alpha,\beta)$) the received signal *amplitude* (e.g., voltage) will fade below 6 dB.

ii). Repeat (i) for $\kappa = 0$. *Note:* $\mathcal{I}_0\,(0) = 1$. Comment on the outage performance with proper reasoning.

**Q. V. Valid or Invalid (4 points)** *Note: Write down Valid/Invalid. Justify your response.*

1) The eccentricity of an elliptical orbit can be expressed as $e = \frac{\mathcal{R}_a-\mathcal{R}_p}{\mathcal{R}_a+\mathcal{R}_p}$, where $\mathcal{R}_p$ is the perigee distance and $\mathcal{R}_a$ is the apogee distance.
2) An electronic telemetry system in a satellite comprises three motion sensors (for three orthogonal directions), a transmitter, and a signal conditioner. Suppose each motion sensor fails with probability 0.01%. Further, the signal conditioner and transmitter fail with probability 0.1%. The probability of satellite telemetry system failure $p_f$ is approximately 2.3%. *Note:* Assume that component failures are statistically independent. Failure of any component leads to the failure of the telemetry system.
3) Consider a five spot beam satellite system. Let these spot beams are denoted by $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_5$. Suppose the the beams $\mathcal{B}_1$, $\mathcal{B}_3$ and $\mathcal{B}_5$ reuse the same frequency (say $f_1$). The remaining two beams use the same frequency (say $f_2$). Further, the system also uses dual polarization. Compared to the system without frequency reuse and polarization, this multi-beam, dual polarization satellite system could achieve capacity by a factor of (approximately) 3.33.
4) Suppose a synchronous DS−CDMA system uses pseudorandom sequences of period $2^9 - 1$. The processing gain is approximately 27.1 dB.