# COMPREHENSIVE EXAMINATION 

Instructor: B. Sainath, Electrical and Electronics Eng. Dept., BITS Pilani. Course: Advanced Satellite Communication (EEE G522)

DATE: Dec. $5^{\text {th }} 2016$ (AN) MAX. POINTS: 40 MAX. TIME: 3 hrs .

## Important instructions:

- Exam is fully closed book. Calculators are allowed.
- Each question/problem is marked with the number of points assigned to that question/problem.
- You can answer questions in any order. But, provide answers to all subparts within a question at one place.
- Give all your answers with proper units.
- In all sketches/illustrations, please include labels and figure caption. Incomplete figures will not get full credit.
Q. 1. [GPS: Clock offset and Earth radius computation]

Suppose that a GPS receiver is located at the south pole, which is the southernmost point on the Earth. The instantaneous coordinates of four satellites (in kilometer) are as follows. $\mathcal{S}_{1}(0,-13280.5,-23002.5), \mathcal{S}_{2}(0,13280.5$, $-23002.5), \mathcal{S}_{3}(13280.5,0,-23002.5)$, and, $\mathcal{S}_{4}(0,0,-26561.0)$. The corresponding delay times are $t_{1}=t_{2}=t_{3}=$ 0.12102731 second, and, $t_{4}=0.11738995$ second. Assuming velocity of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}$, write down range equations. Determine clock offset and the radius of earth at the south pole. (Hint: Start with initial estimate of earth radius as 6378 kilometer and use iterative procedure to determine solution to a precision of 100 cm .) $[2+4+4$ points $]$

Ans. To determine clock offset and radius of earth at the south pole: Compute $P R_{1,2,3}=T_{1,2,3} c=36283.075 \mathrm{KM}$ and $P R_{4}=T_{4} c=35192.622 \mathrm{KM}$. Coordinates of GPS receiver located at south pole $\left(x_{r}, y_{r}, z_{r}\right)=\left(0,0,-z_{p}\right)$.

Since three time delays are same, we get the following range equations:

$$
\begin{aligned}
13280.5^{2}+\left(-23002.5-z_{r}\right)^{2} & =(36283.075-\tau c)^{2}, \\
\left(-26561.0-z_{r}\right)^{2} & =(35192.622-\tau c)^{2},
\end{aligned}
$$

where $\tau$ is clock offset or bias, velocity of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
Using initial value of $z_{p}=6378 \mathrm{KM}$, we get $\tau c=15005.253 \mathrm{KM}$ and $\tau c=15009.622 \mathrm{KM}$. Since the values of $\tau c$ are different, go for next iteration. Following iterative procedure, we get $z_{p} \approx 6358 \mathrm{KM}$, which gives the desired precision. Computing for clock offset, we get $\tau=50.0000 \mathrm{~ms}$.
Q. 2. [VSAT and its Link Budget]
a). With the help of an illustration, state the difference between inbound and outbound links ? [2 points]

Various quantities pertaining to the link budget are given in the following table.

| Quantity | Value |
| :---: | :---: |
| $\underline{\text { UPLINK }}$ |  |
| Frequency (GHz) | 14 |
| Earth station EIRP (dBW) | 67.1 |
| Path loss (dB) | 208.5 |
| Transponder figure of merit $(\mathrm{dB} / \mathrm{K})$ | 2.5 |


| Quantity | Value |
| :---: | :---: |
| DOWNLINK |  |
| Frequency $(\mathrm{GHz})$ | 12 |
| EIRP (dBW) | 38 |
| Path loss $(\mathrm{dB})$ | 206.6 |
| Figure of merit $(\mathrm{dB} / \mathrm{K})$ | 21 |

Compute the following : (Boltzmann's constant $=1.38 \times 10^{-23}\left(J /{ }^{\circ} \mathrm{K}\right)$.)
b). Uplink CNR [2.5 points] c). Downlink CNR [2.5 points]
d). If carrier power-to-interference plus intermodulation ratio is 81 dB , and if overall losses amount to 0.9 dB , what is the resultant CNR (in dB )? [1 point]

Ans. a). The difference between the inbound link and the outbound link is illustrated in the Figure 1 .


Fig. 1: Inbound anb outbound links in VSAT.
b). Uplink CNR $=89.7 \mathrm{~dB} . \mathrm{c})$. Downlink $\mathrm{CNR}=81.0 \mathrm{~dB} . \mathrm{d})$. Effective $\mathrm{CNR}=77.7 \mathrm{~dB}$, accounting overall losses of 0.9 dB , we get 76.8 dB .
Q. 3. [TDMA system parameters]
a). Let $B_{f}$ denote the number of traffic bursts plus the number of reference bursts in the frame, $b_{h}$ be the number of bits in the header of each burst, and, $g_{i}$ be the equivalent duration in bits of the guard interval. Note that guard interval is required between two successive bursts. Furthermore, let $T_{F}$ denote the frame length and ' R ' denote the frame rate in bits per second. Assuming single carrier operation and that the frame contains two reference bursts, derive the expressions for the frame efficiency and th information bit rate. [4 points]
b). Let the $r_{\mathrm{VCH}}$ be the PCM encoded voice channel data rate. Derive expression for the maximum number of PCM voice channels carried in a frame. [2 points]

Ans. a). (i). TDMA frame efficiency $\eta=1-\frac{B_{f}\left(b_{h}+g_{i}\right)}{R T_{F}}$. (ii). Information rate $=\eta R$ bps.
Note: If you consider ' $B$ ' as the number of traffic bursts and two reference bursts, $\eta=1-\frac{\left(B_{f}+2\right)\left(b_{h}+g_{i}\right)}{R T_{F}}$. This quantity is nothing but fraction of information bits transmitted within a frame.
b). Maximum number of PCM voice channels $=\left\lfloor\frac{\eta R}{r_{\mathrm{VCH}}}\right\rfloor$.

## Q. 4. [GPS Ranging Codes]

a). Describe the two ranging codes in the GPS, namely, C/A code and P code with the help of an illustration.
b). What are the chip rates of the two ranging codes, namely, the $\mathrm{C} / \mathrm{A}$ code and the ' P ' code in bps?. Compute the range inaccuracies of the GPS system using (i) C/A code (ii) P code. Assume the velocity of electromagnetic wave as $2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}$. (Give your answers in meters.) [ $2+3$ points]

Ans. a). GPS ranging codes are described below with an illustration.


Fig. 2: GPS carrier and ranging codes.
(i). Range inaccuracy for $\mathrm{C} / \mathrm{A}$ code $=\frac{2.99792458 \times 10^{8}}{1.023 \times 10^{6}}=293.0523 \mathrm{~m}$.
(ii). Range inaccuracy for ' P ' code $=\frac{2.99792458 \times 10^{8}}{10.23 \times 10^{6}}=29.3052 \mathrm{~m}$.
Q. 5. [Redundancy in Satellite System]
a). With illustrations, briefly explain the four forms of redundancy connections in satellites. [4 points]

Ans. Various redundancy connections in satellite subsystems are illustrated below.


Fig. 3: Redundancy connections in satellite subsystems.

Redundancy connections enhance satellite system reliability. For example, series connections used in solar cell arrays, parallel connections to provide redundancy in high power amplifiers.
b). Every subsystem of earth station (ES) except antenna subsystem employs some form of redundancy to enhance reliability. Assuming that failures occur randomly, the probability of the hardware being operated larger than the time interval is given by the exponential distribution as $R(t)=\exp (-\lambda t)$, where $\lambda$ is the average failure rate. Furthermore, mean time to failure (MTTF), denoted by $\mu$, is given by $\mu=\int_{0}^{\infty}-t d R(t)$.
(i). Express $\lambda$ in terms of MTTF.
(ii). If 'L' independent subsystems are connected in series, show that, the overall MTTF $\mu_{s}=\frac{1}{\sum_{i=1}^{L} \lambda_{i}}$, where $\lambda_{i}$ is the average failure rate of each subsystem.
(iii) When 'L' identical subsystems are connected in parallel, derive expression for the overall MTTF $\mu_{p}$ in integral form. $[1+2+2$ points $]$
b). From the given MTTF formula, we have

$$
\mu=\lambda \int_{0}^{\infty} t \exp (-\lambda t) d t
$$

Using integration by parts, it is easy to show that $M T T F=\frac{1}{\lambda}$. Therefore, $\lambda=\frac{1}{\text { MTTF }}$.
(b). Since subsystems are independent, we have $R_{s}(t)=\exp \left(-\left(\lambda_{1}+\ldots+\lambda_{L}\right) t\right)$. Let $\lambda_{s}=\lambda_{1}+\ldots+\lambda_{L}$. Using the result in (a), we can show that $\mu_{s}=\frac{1}{\sum_{i=1}^{L} \lambda_{i}}$.
(c). In this case, we have $R_{p}(t)=1-\left(1-\exp \left(-\lambda_{1} t\right)\right)\left(1-\exp \left(-\lambda_{2} t\right)\right) \ldots\left(1-\exp \left(-\lambda_{L} t\right)\right)$. Since the ' $L$ ' subsystems are identical, $R_{p}(t)=1-(1-\exp (-\lambda t))^{L}$.

Therefore, $\mu_{p}$, in integral form, is given by

$$
\mu=L \lambda \int_{0}^{\infty} t \exp (-\lambda t)(1-\exp (-\lambda t))^{L-1} d t
$$

Q. 6. True or False (2 points)

Note: Each question carries $\frac{1}{2}$ point. Just indicate 'T/F' (' $T$ ' for True statement; ' $F$ ' for false statement.).
(a) A mixer has a noise figure of 9 dB . Assuming $290^{\circ} \mathrm{K}$ as reference temperature, its noise temperature is approximately $2014^{\circ}$ K. Ans. TRUE
(b) In a DS-CDMA MSAT system, interuser interference dominates AWGN when $\frac{E_{b}}{N_{0}} \ll \frac{E_{b}}{N_{0}}$. Ans. TRUE
(c) The spectral efficiency of DS-CDMA MSAT network is directly proportional to CDMA spread bandwidth. Ans.

## FALSE

(d) Satellite coverage area depends on its orbital period. Ans. FALSE

