Birla Institute of Technology and Science Pilani (Rajasthan)

EEE G612 Coding Theory and Practice
First Semester 2022-23
Date : 04/11/2022 **Mid Semester Exam** **Max. Marks: 60**

Name: __________________________________ ID No. __________________ **Time: 90 min**

Instructions:

1) This is a closed book exam. Only 1 A4 size handwritten cheat sheet is allowed.

2) Show all the steps clearly. If I cannot interpret it, I cannot grade it.

3) Sharing of calculators or any other stationary is not permitted during the exam.

Q.1) Consider the MIMO channel wherein the singular value matrix obtained from the SVD of channel gain matrix **H** is given as,

$$\sum = \begin{matrix} \sqrt{52} & 0 & 0 \\ 0 & \sqrt{13} & 0 \\ 0 & 0 & 2 \end{matrix}$$

Considering a transmit power of $\mathrm{P} = -1.25\,\mathrm{dB}$ and noise power $\sigma_n^2 = 3\,\mathrm{dB}$, compute the MIMO capacity and optimal power allocation. **(10)**

Q.2) Consider a Markov chain whose probability transition matrix is given as,

$$\boldsymbol{P} = \begin{matrix} 0 & 1 & 0 \\ \alpha & 0 & 1-\alpha \\ 1 & 0 & 0 \end{matrix}$$

(i) Draw the state transition diagram
(ii) Find the stationary distribution
(iii) For what value of α will the entropy rate be maximum? Also what is the maximum entropy rate of this Markov chain? **(2+ 5+ 5 = 12)**

Q.3a) Obtain the code for binary string 00000011010100000110101 using a suitable source coding scheme? Justify the choice of source coding scheme that you use. **(8)**

Q.3b) Consider a random variable X which takes 6 values $\{A, B, C, D, E, F\}$ with probabilities $(0.5, 0.25, 0.1, 0.05, 0.05, 0.05)$, respectively. Construct a quaternary (4-ary) Huffman code for this random variable, and find the efficiency of the code. **(5)**

Q.4) Let C be the linear code of block length 5 in which the fourth bit of the code words (from left) checks the parity of the first two bits, and the last bit (from left) is the overall parity check.
(i) Construct the standard array for this code
(ii) Determine if any error has occurred during the transmission if the received code word is 11111. If yes, then correctly decode the received code word using standard array decoding.
(iii) How can the correct code word be retrieved from the received code word? **(10+3+2 = 15)**

Q.5) Derive the differential entropy for the following:
(i) Normal random variable X with mean = 0, and variance = σ^2. Use this expression to evaluate the differential entropy of another random variable $Y = X_1 + X_2$, where X_1, X_2 are normally distributed with 0 mean and variances σ_1^2, σ_2^2, respectively.
(ii) Exponential random variable X with mean = $1/\lambda$ **(7+3 = 10)**

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