Birla Institute of Technology and Science Pilani (Rajasthan)

EEE G612 Coding Theory and Practice
First Semester 2023-24

Date : 09/10/2023 **Mid Semester Exam** **Max. Marks: 60**

Name: _____ID No. _____ **Time: 90 min**

Instructions:

1) This is a closed book exam. Only 1 A4 size handwritten cheat sheet is allowed.

2) Show all the steps clearly. If I cannot interpret it, I cannot grade it.

3) Sharing of calculators or any other stationary is not permitted during the exam.

Q.1) Consider the Lempel Ziv encoding for quaternary data (symbols: 0,1,2,3). Encode the following data: 1 3 3 0 0 2 0 2 1 1 1 3 0 0 0 0 2 2 1 2 2 2 3. What is the compression ratio obtained? Does this scheme actually cause any compression? Justify. **[8]**

Q.2) Consider a MIMO channel wherein the singular value matrix obtained from the SVD of **H** matrix is given as,

$$\sum = \begin{matrix} 0.9719 & 0 & 0 \\ 0 & 0.2619 & 0 \\ 0 & 0 & 0.0825 \end{matrix}$$

Assume that **H** is known both to the transmitter and receiver. The system has $\frac{P}{\sigma^2} = 20\ dB$, and a bandwidth of $B = 100$ kHz. Find the optimal power allocation $(\frac{P_i}{P})$ and the total capacity. **[10]**

Q.3) Consider a Markov chain whose probability transition matrix is given as,

$$\boldsymbol{P} = \begin{matrix} 0 & 1 & 0 \\ \alpha & 0 & 1-\alpha \\ 1 & 0 & 0 \end{matrix}$$

(i) Draw the state transition diagram
(ii) Find the stationary distribution
(iii) For what value of α will the entropy rate be maximum? Also what is the maximum entropy rate of this Markov chain? **[2+ 5+ 5 = 12]**

Q.4) In a repetitive BSC, input symbols are $x_1 = 00, x_2 = 11$, and the output symbols are $y_1 = 00, y_2 = 01, y_3 = 10$ and $y_4 = 11$. Symbols x_1 and x_2 are equiprobable. The transition probabilities for received bit b_j when the transmitted bit is b_i are $P(b_j = 1 | b_i = 0) = P(b_j = 0 | b_i = 1) = p$ and $P(b_j = 1 | b_i = 1) = P(b_j = 0 | b_i = 0) = 1 - p = q$.

(i) Create the channel graph and write the channel matrix **[3]**
(ii) Find $H(y), H(y|x)$, and hence $I(x; y)$ **[4+4+4 = 12]**

Q.5) Let C be the linear code in $GF(2)$, having block length 5, in which the fourth bit of the codewords (from left) checks the parity of the first two bits, and the last bit (from left) is the overall parity check, i.e., if $c_i = \{x_1 x_2 x_3 x_4 x_5\}$ is a codeword, then $x_4 = x_1 + x_2$, and $x_5 = x_1 + x_2 + x_3 + x_4$.
(i) Construct the standard array for this code
(ii) Determine if any error has occurred during the transmission if the received code word is 00100. If yes, then decode the received code word using standard array decoding.
(iii) How can the decoded code word be retrieved from the received code word? **[10+3+2 = 15]**

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