Birla Institute of Technology and Science Pilani (Rajasthan)

EEE G612 Coding Theory and Practice
First Semester 2023-24

Date : 07/12/2023 **Comprehensive Exam** **Max. Marks: 80**

Name: ________________________________ ID No. ________________ Time: 3 hrs

Instructions:

1) This is a closed book exam. Only 1 A4 size handwritten cheat sheet is allowed.

2) Show all the steps clearly. If I cannot interpret it, I cannot grade it.

Q.1) Consider that the rate $2/3$ trellis-coded system is used over a binary symmetric channel (BSC). The trellis diagram indicating the waveform assignments and their corresponding bit representation using certain coding scheme is shown in Fig. 1. Assume that the initial encoder state is the 00 state. At the output of the BSC, the sequence $Z = (111001101011\ rest\ all\ "0"s\ )$ is received.

[8+7 = 15 Marks]

(i) Find the maximum likelihood path through the trellis diagram and determine the first 6 decoded information bits. If a tie occurs between any two merged paths, choose the upper branch entering the particular state.

(ii) Determine if any channel bits in Z had been inverted by the channel during transmission, and if so, identify them.

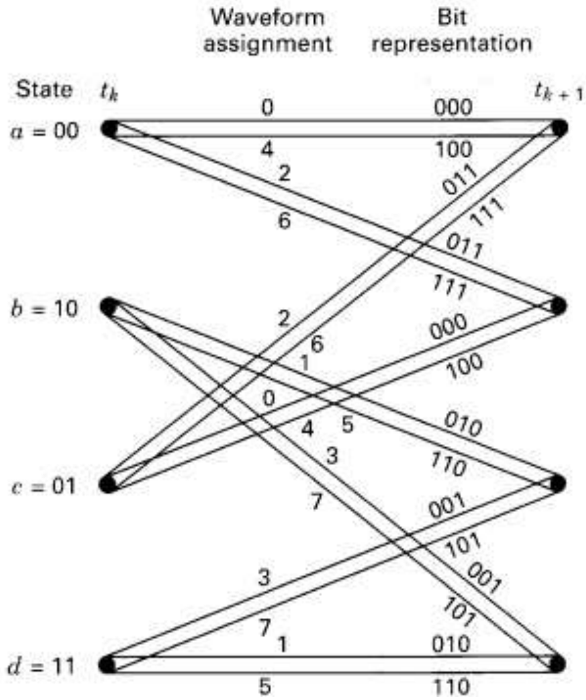

Fig. 1

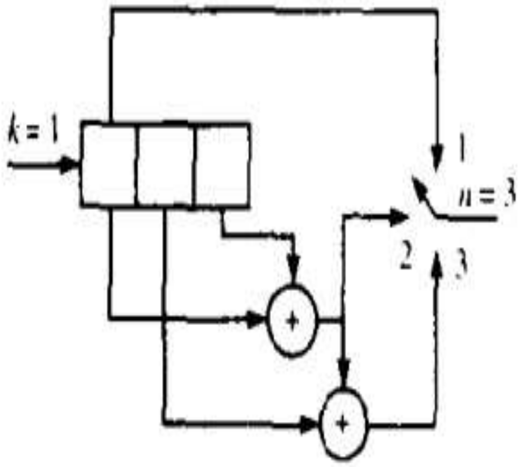

Fig. 2

Q.2a) The block diagram of a binary convolutional code is shown in Fig. 2. Consider that the first flip flop is used for storing the input, while rest are the part of encoding logic circuit. **[3+5+1 = 9 Marks]**

(i) Draw the state diagram for the code.

(ii) Find the transfer function of the code $T(D, L, I)$, where D, L, I hold their usual meanings

(iii) What is the minimum free distance of the code?

Q.2b) The elements of generator polynomial matrix of rate ½ convolutional codes are given . Determine which of the following are catastrophic and why? **[2+2+2 = 6 Marks]**

(i) $g_1(X) = 1 + X + X^2,\ \ g_2(X) = 1 + X + X^3 + X^4$

(ii) $g_1(X) = 1 + X^3 + X^4,\ g_2(X) = 1 + X + X^2 + X^4$

(iii) $g_1(X) =\ 1 + X + X^3 + X^4, g_2(X) = 1 + X^2 + X^4$

P.T.O

Q.3a) A memoryless source emits messages m_1 and m_2 with probabilities 0.8 and 0.2. Find the Huffman binary code for the third-order extension of the source, and the code efficiency. **[10 Marks]**

Q.3b) Consider a channel with binary inputs that has both erasures and errors. Let the probability of error be ϵ and the probability of erasure be α. The channel graph is as shown in Fig. 3.

[8+2 = 10 Marks]

(i) Find the expression for the capacity of this channel
(ii) Specialize (i) to the case $\alpha = 0$ and the case $\epsilon = 0$. Identify the corresponding channel types.

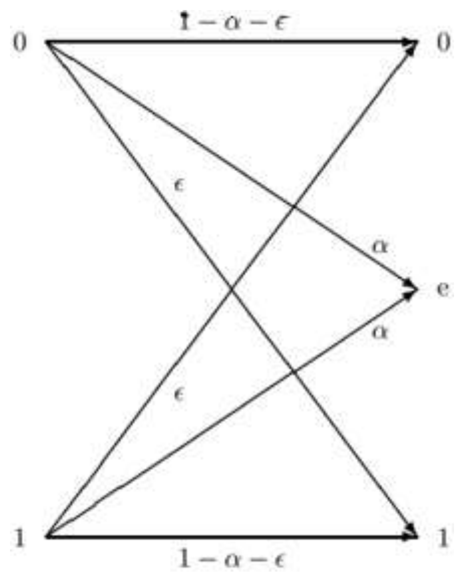

Fig. 3

Q.4) For a $(11,7)$ Hamming code, determine the following. **[6+2+2 = 10 Marks]**
(i) Generator matrix G, Parity check matrix H
(ii) Using G, encode the input 1001101 as a Hamming code
(iii) If the received code word is 10010100101, determine the error location using H.

Q.5) Consider the $(15,5)$ triple error correcting BCH code. Assume $GF(2)$ and $GF(16)$ as the base and the extension field, respectively, such that α is the primitive element and the extension field is obtained from base field using the primitive polynomial $1 + X + X^4$. Additionally, you may use $\alpha^{10} = 1 + \alpha + \alpha^2$ for simplifications as necessary. If the received code word polynomial is $r(X) = X^3 + X^5 + X^{12}$, find the following. **[6+14 = 20 Marks]**

(i) What is the syndrome for this code. The components of syndrome tuple should be strictly expressed as a single power of the primitive element.
(ii) Determine the error locator polynomial $\sigma(X)$. How many errors have actually occurred in the received code word during the transmission?

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