Mid-Semester Test

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Course No./Title : EEE G613/Advanced Digital Signal Processing

DATE: Oct. 31th, 2022. Test duration: 90 Mins. Max. marks: 25.

(**Note:** Exam is fully closed book. Answer all the questions. Incomplete figures will not get full credit. Use the same notation given in the questions. In all your answers, show important intermediate steps. Highlight final answers in rectangular boxes. Simplify your answers to the extent possible. *Answer all the subparts of a question at one place. Overwritten responses will not be rechecked.*)

Q.1. [5 points] i). Write down the four basic building blocks of "TDM $\Rightarrow$ FDM $\Rightarrow$ TDM" multirate transmultiplexer (TMUX). (*Note:* Write the names of the blocks only.) [1 point]

ii). We can view the TMUX as an LTI system with transfer function $\Psi(z)$. Let $X(z)$ and $\widehat{X}(z)$ denote the input and the output, respectively. In the $z-$ domain, how $x[n]$ and $\widehat{x}[n]$ related? Suppose that $\widehat{X}_k(z) = 9z^{-4} + 18z^{-5} + 9z^{-6}$ and $X_k(z) = (1 + z^{-1})^2$. For all k, express $\psi_{kk}(z)$. Comment on the nature of the TMUX. [3 points]

iii). To avoid amplitude distortion, what is the condition on $\psi_{kk}(z)$? Furthermore, what is the condition on $\psi_{kk}(z)$ to avoid phase distortion? [1 point]

Q.2. [5 points] Suppose that $H(z) = \frac{1}{1-2(\rho\cos\phi)z^{-1}+\rho^2 z^{-2}}$, with $\rho > 0$, ϕ real. *Answer the following:*

i). Determine the poles and Comment. Specify the poles in the most simplified form. [1 point]

ii). Recall the type$-$I polyphase representation. Express the $E_0(z^2)$ and $E_1(z^2)$) in terms of $H(z)$ and $H(-z)$. Using these relations, determine $E_0(z)$ and $E_1(z)$. [1 + 2 points]

iii). Let $\phi = 0$. Determine $\left|\widetilde{H}(e^{j\omega})\right|$ (in dB scale) at $\omega = \pi$. [1 point]

Q.3. [5 points] Suppose that a filter has the impulse response

$$h[n] = \begin{cases} 1, & 0 \le n \le (L-1), \\ 0, & \text{elsewhere}, \end{cases}$$

where L is a positive integer ≥ 2. *Answer the following:*

i). Determine the frequency response by explicitly specifying the magnitude response and the phase response. Sketch the magnitude response for $L = 5$ for $0 < \omega \leq 2\pi$. Comment on the nature of the filter. [2 + 1 + 1 points]

ii). Group delay is defined as $\tau(\omega) = -\frac{d}{d\omega}\left[\arg\{\mathcal{H}(e^{j\omega})\}\right]$. Determine the group delay. [1 point]

Q.4. [5 points] Consider an implementation of uniform DFT filter band for $[H_0(z)\ H_1[z]]$. Let $H_0(z) = 1 + 2z^{-1} + 4z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5}$ and $H_1(z) = H_0(-z)$. Refer to the Fig. 1. *Answer the following:*

i). Determine $X_0(z)$ and $X_1(z)$. [1 point]

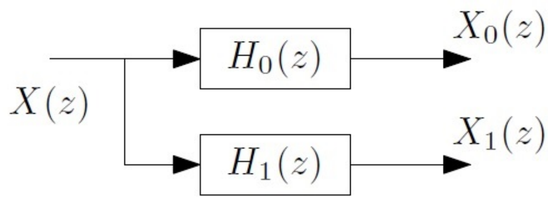

Fig. 1. Pertaining to Q.4.

ii). Neatly draw a schematic of the DFT analysis filter bank. *Note:* Explicitly show input, polyphase components, IDFT matrix, and outputs. [4 points]

Q. 5: Statements justification (5 points)

Note: Write whether the following are valid or invalid. Justify sufficiently and precisely.

1) Let $H(z) = \sum_{n=0}^{N} h[n]z^{-n}$ is a linear phase FIR filter (type−I). Let $g[n] = e^{j64\pi}\delta[n-64] - e^{j2\pi n}h[n]$. We have type−II linear phase FIR filter denoted by $G(z)$.
2) Consider the following two polyphase components in a type−I polyphase representation: $E_0(z) = 1 + 9z^{-1}$ and $E_1(z) = -9z^{-0.5}$. We have $\frac{1}{2}(H(z) + H(-z)) = 1$.
3) Consider the power spectrum of a scalar random process $S_Y(e^{j\omega}) = 2\pi\left(c_0\delta(\omega-\omega_0) + c_1\delta(\omega-\omega_1)\right)$ where $0 \le \omega < 2\pi$, $c_0 = \frac{1}{\sqrt{2}}(1+j)$ and $c_1 = \frac{1}{\sqrt{2}}(1-j)$. At $k = 0$, the autocorrelation function value $R_Y[0] = \sqrt{2}$.
4) Suppose that $w[n]$, a zero mean AWGN process with power spectrum $\Phi_{WW}[e^{j\omega}] = \frac{N_0}{2}, |\omega| \le \pi$, is transmitted through an ideal lowpass filter (LPF). The output of the ideal LPF is given by
$$\Phi_{WW}[e^{j\omega}] = \begin{cases} \frac{N_0}{2}, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \le \pi. \end{cases}$$
If the average output power is $0.45N_0$, $\omega_c = 0.9\pi$.
5) Let $\mathcal{H}\{.\}$ denote the ideal Hilbert transformer. We have $\sum_{n=-\infty}^{\infty} x[n]\mathcal{H}\{x[n]\} = 0$.

□ END OF QUESTION PAPER □