Mid-Semester Test

Instructor-in-charge: B. Sainath, 2210-A, Electrical and Electronics Eng. Dept., BITS Pilani.

Course No./Title : EEE G613/Advanced Digital Signal Processing

DATE: Oct. 13th, 2023. Test duration: 90 Mins. Max. marks: 25.

(**Note:** Exam is fully closed book. Answer all the questions. Incomplete figures will not get full credit. Use the same notation given in the questions. In all your answers, show important intermediate steps. Highlight final answers in rectangular boxes. Simplify your answers to the extent possible. *Answer all the subparts of a question at one place. Overwritten responses will not be rechecked.*)

Q.1. On DFT and Peak to average power ratio [4 points]

Consider a four-point sequence $\mathbf{d} = [1 \;\; 1 \;\; 1 \;\; 1]^t$. *Answer the following:*

i). Determine and write down the DFT matrix. Denote it by Q.

ii). Using the DFT matrix Q, determine the DFT of the discrete-time sequence $\mathbf{d}$.

iii). Compute $\frac{\max \; |d[n]|^2}{\mathbf{E}[|d[n]|^2]}$ in dB.

Q.2. Binary Hypotheses Testing and Detection [5 points]

Consider the following detection problem using binary Hypothesis testing.

$$\mathcal{H}_0 : \;\; \mathcal{R} = -1 + W,$$

$$\mathcal{H}_1 : \;\; \mathcal{R} = 1 + W,$$

where the noise random variable $W \sim \mathcal{U}[-\frac{b}{2}, \; \frac{b}{2}]$. Assume that the symbols $+1$ and -1 are equally likely. *Answer the following:*

i). Note that W is uniformly distributed with mean 0 and variance 1. Determine the value of b.

ii). Consider the following detector: $\mathcal{R} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma$, where γ is the threshold, which is taken as 0. *Answer the following:*

Comment on the following error probabilities: $\mathcal{P}r\left(\mathcal{R} > 0|\mathcal{H}_0\right)$ and $\mathcal{P}r\left(\mathcal{R} < 0|\mathcal{H}_1\right)$.

Determine: $\frac{1}{2}\mathcal{P}r\left(\mathcal{R} > 0|\mathcal{H}_0\right) + \frac{1}{2}\mathcal{P}r\left(\mathcal{R} < 0|\mathcal{H}_1\right)$.

Q.3. On Differencer System Output Random Process [5 points]

Let $X[n]$ denote a discrete-time white Gaussian noise process with $\sigma_X^2 = 1$. Suppose $X[n]$ is applied as input to a difference system to generate output random process $Y[n] = X[n] - X[n-1]$. *Answer the following:*

i). The probability density function of the $\mathbf{Y} \triangleq [Y[0], Y[1]]^t$, denoted by $Y \sim \mathcal{N}(\mu_{\mathbf{Y}}, K_{\mathbf{Y}})$. Determine the mean vector $\mu_{\mathbf{Y}}$ and the covariance matrix $K_{\mathbf{Y}}$.

ii). Are the samples $[Y[0]$ and $Y[1]]$ statistically independent? Justify your response.

Q.4. On Frequency Response and Impulse Response [6 points]

Consider the following frequency response:

$$H_d[e^{j\omega}] = (1 - 2u[\omega])\, e^{j\left(\frac{\pi}{2} - \omega\tau_d\right)}, \quad -\pi < \omega < \pi,$$

where $u[\omega]$ is the unit step function and τ_d is the delay. *Answer the following:*

i). Determine and write down the mathematical forms of $\left|H_d[e^{j\omega}]\right|$ for all ω (magnitude response) and $\angle H_d[e^{j\omega}]$ for all ω (phase response).

ii). Neatly sketch the phase response for the interval $-\pi < \omega < \pi$. Clearly label the x-axis and y-axis. Indicate the phase(s) at $\omega = 0$. Comment on the DT system.

iii). Determine $h_d[n]$ for $n \neq \tau_d$ and $n = \tau_d$.

Q. 5: Statements justification (5 points)

Note: Write whether the following are valid or invalid. Justify sufficiently and precisely.

1) Consider the DC level detection problem:

$$\mathcal{H}_0 : \; X[n] = W[n], \; n = 0, 1, \ldots, 9.$$
$$\mathcal{H}_1 : \; X[n] = A + W[n], \; n = 0, 1, \ldots, 9.$$

Define SNR $= \frac{A^2}{\sigma^2}$. Suppose the deflection constant is 100. The SNR is equal to 10 dB.

2) Consider the design of the Butterworth IIR lowpass filter with the following specifications: $\Omega_p = 0.2\pi$ and $\Omega_s = 0.3\pi$. Further,

$$|H_a(j\Omega_p)|^2 = 0.81, \quad |H_a(j\Omega_s)|^2 = 0.01.$$

The lowpass filter order is 9.

3) Consider the system function $H(z) = \frac{z^{-1} - 0.45}{1 - 0.45z^{-1}}$. This function represents an all-pass system.

4) Consider a causal LTI system characterized by $H(z) = \frac{1}{1 - z^{-1} + z^{-2}}$. The system is stable.

5) Consider the following discrete-time correlation function. $R_Y[k] = \frac{1}{2} + \frac{1}{4}\delta[k-1] + \delta[k] + \frac{1}{4}\delta[k+1]$. The PSD is given by $\frac{3+\cos(2\pi f)}{2}$.

□ END OF QUESTION PAPER □