Birla Institute of Technology and Science, Pilani

Advanced Wireless Communications (EEE G614)

Mid-Semester Examination, First Semester 2023 – 24

Time: 90 minutes | Maximum Marks: 60

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1. Answer the following in the context of the 5G wireless communications system.
   (a) Describe the 5G NR synchronization procedure in detail explaining the significance of SSS, PSS, PBCH, and PDSCH. **(8)**
   (b) Give the 5G NR frame structure. **(4)**

2. A binomial random variable X with second moment 5.98 is approximated as a Poisson random variable Y satisfying $2\Pr[|Y-1|<2] = 5\Pr[Y=2]$.
   (a) Find the c.f.s $\Psi_Y(jw)$ and $\Psi_X(jw)$. **(4+4)**
   (b) Calculate the c.d.f. value $F_Y(3)$. **(4)**

3. A bivariate Gaussian p.d.f. is given by

$$f_{\underline{X}}(\underline{x}) = \frac{1}{\alpha\,\pi}\exp\left\{-\frac{[4x_1^2+5x_2^2+4x_1x_2-80x_1-40x_2+\beta]}{160}\right\}, -\infty < x_1, x_2 < \infty,$$

where $\underline{X} = [X_1, X_2]^T$, $\underline{x} = [x_1, x_2]^T$, and $\underline{X} \sim \mathcal{N}(\underline{\mu_X}, \underline{K_X})$. The vector $\underline{X}$ is transformed to another vector $\underline{Y} = [Y_1, Y_2]^T$ having a $\mathcal{N}(\underline{\mu_Y}, \underline{K_Y})$ distribution with c.f.

$$\Psi_{Y_1,Y_2}(jw_1, jw_2) = \exp\left(2[jw_1+1]^2 + 5\left[jw_2 - \frac{2}{5}\right]^2 - 2w_1w_2 - \frac{14}{5}\right)$$

by an affine transformation $\underline{Y} = \underline{AX} + \underline{b}$, where $\underline{A}$ is a lower triangular matrix with positive diagonal elements.
   (a) Calculate α, β, $\mathbb{E}[X_1^3]$, and $\mathbb{E}[X_1X_2^3]$. **(12)**
   (b) Find A and b. **(12)**

4. A Nakagami distributed random variable Y with parameter m ($m > 0$) and second moment Ω has the following properties

$$\Psi_{Y^2}(jw) = \frac{1}{(1-4jw)^m}, \quad \mathbb{E}[Y^4] = 192.$$

Calculate m and Ω. **(12)**

Some Formulae

- If $\underline{X} \sim \mathcal{N}(\underline{\mu}, \underline{K})$ and $\underline{X}$ is $L \times 1$, then

$$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)^{L/2}\{\det(\underline{K})\}^{1/2}}\exp\left\{-\frac{1}{2}(\underline{x}-\underline{\mu})^T\underline{K}^{-1}(\underline{x}-\underline{\mu})\right\}, x \in \mathcal{R}^L,$$

$$\Psi_{\underline{AX}+\underline{b}}(j\underline{\omega}) = \exp\left\{j\underline{\omega}^T\underline{A}\underline{\mu} + j\underline{\omega}^T\underline{b} - \frac{1}{2}\underline{\omega}T\underline{AKA}^T\underline{\omega}\right\}$$

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$\Psi_{X^2}(j\omega) = \frac{\exp\left\{\frac{j\omega\mu^2}{1-2j\omega\sigma^2}\right\}}{(1-2j\omega\sigma^2)^{1/2}}$$

- Gamma distribution with parameter m and mean Ω:

$$f_X(x) = \frac{m^m x^{m-1}\exp(-\frac{mx}{\Omega})}{\Gamma(m)\,\Omega^m}, \ x \geq 0, \quad \Psi_X(j\omega) = \frac{1}{(1-j\omega\frac{\Omega}{m})^m}, \quad \mathbb{E}[X^v] = \frac{\Gamma(v+m)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^v$$

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