MID-SEMESTER TEST

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Course: Advanced Digital Communication (EEE G622)

DATE: Mar. 9th 2017 MAX. TIME: 90 Mins. Max. Marks: 30

(*Note*: You may use standard results or formulae. However, state them clearly and precisely.)

Q. 1. a). [On Circularly Symmetric Complex Gaussian (CSCG) Random Variable]

a). Notation: $\mathcal{CN}$ denotes CSCG random variable. $Z \sim \mathcal{CN}(0,1)$. Derive the moment generating function (MGF) of $|Z|^2$. From the MGF, determine the mean and variance of $|Z|^2$. (2+1+1 points)

b). Suppose $Z_1 \sim \mathcal{CN}(0,1)$ and $Z_2 \sim \mathcal{CN}(0,1)$ are $i.i.d.$ random variables, determine the probability density function (PDF) of $|Z_1|^2 + |Z_2|^2$. Sketch the PDF. (3+1 points)

c). (Filtered white noise) Let $W(t)$ be a additive white Gaussian noise process of spectral density σ^2. Let $U = \int_0^1 W(t)\,dt$. Find the probability density function of U. (2 points)

Q. 2. [On Square of Gaussian Q-function]

Recall the proof for $Q(x)$ derived in class. Using similar trick, derive single-integral expression for $Q^2(x)$ in which θ is the variable of integration. [5 points]

Derive an upper bound on $Q^2(x)$. [2 points]

Q. 3. [Matched Filter] A binary baseband communication system transmits one bit for every T_b seconds with $T_b = 4$. The pulse shape used is shown in figure 1.

a). The pulse $s(t)$ is applied as input to the matched filter. Sketch the impulse response $h(t)$ of the matched filter. [2.5 points]

b). Determine the sampled output of the matched filter at t = 4. [2.5 points]

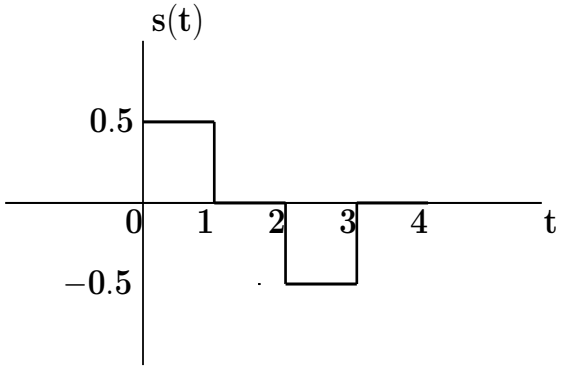

Fig. 1: Pulse shape in Q.3.

Consider the data transmission over baseband channel using polar signaling in which bit '0' is mapped to -1 volt and bit '1' is mapped to $+1$ volt. The line encoded signal is transmitted over baseband channel. The received signal is corrupted by Gaussian noise process with variance σ^2.

c). If the matched filter is used at the receiver, determine the SNR of the sampled output. [2.5 points]

d). Determine the probability of a bit error p_{be} of the matched filter receiver. Express p_{be} in terms of erfc. [2.5 points]

Q. 4. [4-PAM] Consider 4-PAM constellation shown in Figure 2. Consider transmission of symbols through AWGN with two-sided power spectral density $\frac{N_0}{2}$. Assume that all symbols are equally likely.

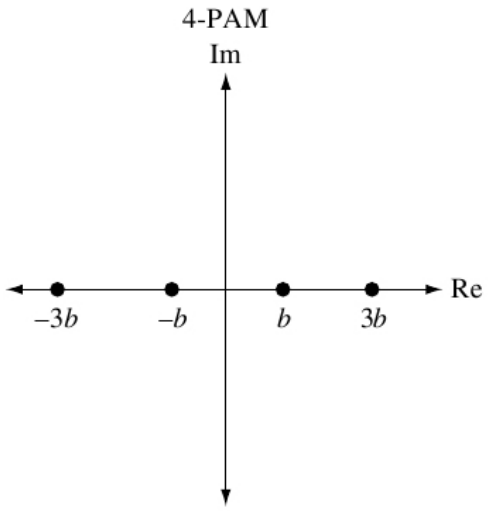

Fig. 2: 4-PAM constellation diagram

a). Compute the value of b if the average transmitted energy per symbol is 1. [1 point]

b). Derive average probability of error (P_e) in terms of erfc function. [2 points]