Birla Institute of Technology & Science, Pilani, Rajasthan
Second Semester 2022-2023
Comprehensie Exam (Open Book)

EEE G622 Advanced Digital Communications

Date: 06-05-2023, Duration: 180 Minutes, Maximum Marks: 40

- There are five questions each of 8 Marks.
- Please make sure to explicitly state any assumptions made when solving the questions.

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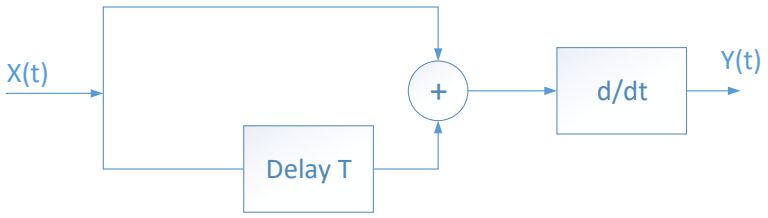
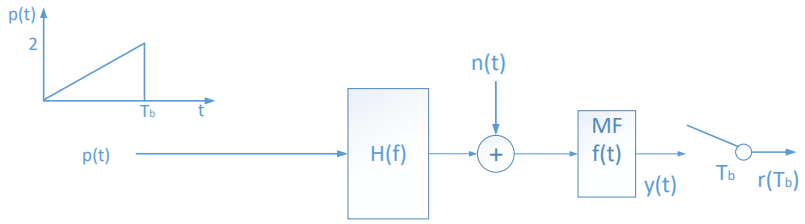
1. (a) Find the PDF of the random variable $Z = \sqrt{X_1^2 + X_2^2}$, if $X_1 \sim N(m, \sigma^2)$ and $X_2 \sim N(0, \sigma^2)$.[**4 Marks**]

   (b) A bandpass input signal $x(t) = \Pi(\frac{t}{T}) \cos 2\pi f_0 t$ is passed through a bandpass system $h(t) = \alpha e^{-\alpha t} u(t) \cos 2\pi f_0 t$. Use the low-pass equivalent of the signals to obtain the output signal $y(t)$. [**2 Marks**]

   (c) An input random process is given as $X(t) = X_1 + X_2$ where $X_1 \sim U(0, 1)$ and $X_2 \sim U(-1, 1)$. Find the PSD of the output process $Y(t)$. Assume the random processes as the WSS. [**2 Marks**]



2. (a) Let signal $p(t)$ is used as OOK modulation over channel $H(f)$ as shown in figure below. The double sided AWGN noise PSD is $N_0/2 = 4$ and the channel $H(f) = (1 + \cos 2\pi f T_b) e^{-j4\pi f}$. Find the MF $f(t)$ and the average BER in terms of Q-function. [**4 Marks**]



(b) Find the SER for a rectangular 64-QAM (in terms of $\frac{E_b}{N_0}$) over AWGN channel with $n(t) \sim N(N_0/2)$ in both in-phase and quadrature-phase components. [**4 Marks**]

3. (a) Derive the SER for an 4-ary FSK with non-coherent detection assuming noise as AWGN with zero mean and unequal variances of the additive noise in orthogonal frequency branches (i.e., i.ni.d. noise). Also sketch the block diagram for the receiver. Use standard notations for the parameters used. [**4 Marks**]

(b) Derive the SER for non-coherent detection for binary OOK over i.i.d AWGN channel. Use standard notations for the parameters used. [**2 Marks**]

(c) Derive the average BER for the DPSK modulation under L-branch selection combining (SC) with i.i.d. Rayleigh fading on each branch. Use standard notations for the parameters used. [**2 Marks**]

4. (a) Assume that symbols are ± 1 and $\pm j$ with equal probability, find the non-decision directed ML phase estimation for the QPSK. Also draw the block diagram of the estimator. [**4 Marks**]

(b) An input signal is given as $x(t) = A\sin(2000\pi t + 3t^2)$. Find the steady-state error for the phase estimation if a PLL is used with the loop filter $\frac{s+a}{s}$. [**4 Marks**]

5. A pulse $p(t) = sinc(at)sinc(bt)$, where $a > b$ is used for digital transmission over bandlimited channels. Use the definition of $sinc(x) = \frac{sin\pi x}{\pi x}$.

(a) Consider that the pulse $p(t)$ is to be used over an ideal real baseband channel with one-sided bandwidth 400 Hz. Find a and b such that the pulse $p(t)$ satisfies the Nyquist first criteria for zero ISI achieving 4-PAM signaling at 1200 bit/s and exactly fills the channel bandwidth. [**4 Marks**]

(b) Consider that the pulse $p(t)$ is to be used over a passband channel spanning the frequencies 2.4–2.42 GHz. Assume that we use 64-QAM signaling at 60 Mbit/s. Find a and b such that the pulse $p(t)$ satisfies the Nyquist first criteria for zero ISI and exactly fills the channel bandwidth. [**4 Marks**]