Birla Institute of Technology and Science, Pilani

Applied Estimation Theory (EEE G641)

Mid-Semester Examination, First Semester 2022 – 23

Time: 90 minutes | Maximum Marks: 35

1. Prove the following, given that Λ denotes the LRT, H_0 and H_1 denote the hypothesis, $\mathbb{E}(\cdot)$ denotes the expectation operator, and Var is the variance.
   (a) $\mathbb{E}\left[\Lambda^n | H_1\right] = \mathbb{E}\left[\Lambda^{n+1} | H_0\right]$ **(2)**
   (b) $\mathbb{E}\left[\Lambda | H_0\right] = 1$ **(2)**
   (c) $\mathbb{E}\left[\Lambda | H_1\right] - \mathbb{E}\left[\Lambda | H_0\right] = \text{Var}\left[\Lambda | H_0\right]$ **(3)**

2. We want to estimate a in Binomial distribution by using n observations.
$$\Pr(r \text{ events}|a) = \binom{n}{r} a^r (1-a)^{n-r}, r = 0, 1, 2, .., n.$$
   (a) Find the ML estimate of a, check if the estimator is unbiased, and compute its variance. **(3+2+2)**
   (b) Is it efficient? **(3)**
   Hint: For a Binomial random variable with parameters (n, p) the mean is np and the variance is $np(1-p)$.

3. We observe x with a Rayleigh PDF given by
$$f_X(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\,\sigma^2}\right), x \geq 0$$
with $H_0 : \sigma^2 = 1$ and $H_1 : \sigma^2 = 3$. We wish to detect the event, increase in the noise level.
   (a) What is the ML detector for this problem? **(2)**
   (b) What is the Neyman-Pearson test that achieves the $P_F = 0.1$, and what is the P_D. **(2+2)**

4. A ternary communication system transmits one of the three amplitude signals $[1, 2, 3]$ with equal probabilities. The independent received signals under each hypothesis are
$$H_1 : Y_k = 1 + N,\ k = 1, 2, ..., K,$$
$$H_2 : Y_k = 2 + N,\ k = 1, 2, ..., K,$$
$$H_3 : Y_k = 3 + N,\ k = 1, 2, ..., K,$$
where N is the AWGN with mean zero and variance σ^2. The costs are $C_{ii} = 0$ and $C_{ij} = 1$ for $i \neq j; i, j = 1, 2, 3$. Determine and plot the decision regions. **(6)**

5. Find the minimum mean-square error and the MAP estimators of X from the observation $Y = X + N$, where X and N are random variables with the density functions $f_X(x) = \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x-1)$ and $f_N(n) = \frac{1}{2}\exp(-|n|)$, respectively. **(3+3)**

End of Paper