X

Birla Institute of Technology and Science, Pilani
Comprehensive Examination, First Semester 2016-2017
MATHEMATICS I (MATH F111)

Date: 08-12-2016 **Max. Time: 3 hours** **Max. Marks: 120**

Important Instructions

- The question paper consists of two parts: **PART-I (CLOSE BOOK)** for 45 minutes and **PART-II (OPEN BOOK)** for 135 minutes.
- Part-II question paper will be given only after submission of Part-I answer sheet.
- Late submission of Part-I is not allowed.
- Do not write anything except the answer on this sheet. Do rough work in the last page of given answer sheet.
- Calculator is not allowed.

PART-I (CLOSE BOOK)

Max. Time: 45 Minutes **Max. Marks: 30**

Name:	**ID. No.:**

Note: In this part, each question carries 3 marks. Please write most appropriate answer in the provided space. Answer written elsewhere will not be evaluated. Overwritting will be considered as unattempted.

1. The set of points (x, y), where the function $f(x, y) = \sqrt{x^2 + y^2}$ is continuous, is **both open and closed** *(open but not closed/closed but not open/both open and closed/neither open nor closed).*

2. The interval of convergence for the series $\sum_{n=1}^{\infty} \sqrt[n]{n}(2x-5)^n$ is $\boldsymbol{2 < x < 3}$**.**

3. An equivalent double integral, with the order of integration reversed, of $\int_{-2}^{0}\int_{0}^{4-y^2} y\,dx\,dy$ is $\boldsymbol{\int_0^4 \int_{-\sqrt{4-x}}^{0} y\,dy\,dx}$**.**

4. The flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the sphere $x^2 + y^2 + z^2 = a^2$ in the outward direction is $\boldsymbol{4\pi a^3}$.

5. For the space curve, $\mathbf{r}(t) = (\sinh t)\mathbf{i} - (\cosh t)\mathbf{j} + t\mathbf{k}$, the unit tangent vector at $t = 0$ is $\boldsymbol{\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}}}$.

6. The area of the triangular surface, cut from the plane $6x + 3y + 2z = 6$ by the coordinate planes, in the first octant is $\boldsymbol{\frac{7}{2}}$.

7. The critical point of the function $f(x, y) = x^2 + xy + 3x + 2y + 5$ is a point of **saddle point.** *(local maxima/local minima/saddle point/no conclusion)*

8. The linearization $L(x, y)$ of the function $f(x, y) = (x + y + 2)^2$ at the point $(1, 2)$ is $\boldsymbol{10x + 10y - 5}$.

9. The flow of the velocity field $\mathbf{F} = 3x\mathbf{i} + 2y\mathbf{j}$ along upper half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ in the xy-plane is $\mathbf{0}$.

10. Let D be the region in the first octant that is bounded below by the cone $\phi = \pi/4$ and above by the sphere $\rho = 3$. The iterated integral for the volume of D in terms of cylindrical coordinates is $\boldsymbol{\int_0^{\pi/2}\int_0^{3/\sqrt{2}}\int_r^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta}$**.**

*****END*****

Y

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Date: 08-12-2016 **Max. Time: 3 hours** **Max. Marks: 120**

Important Instructions

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- Part-II question paper will be given only after submission of Part-I answer sheet.
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- Calculator is not allowed.

PART-I (CLOSE BOOK)

Max. Time: 45 Minutes **Max. Marks: 30**

Name:	**ID. No.:**

Note: In this part, each question carries 3 marks. Please write most appropriate answer in the provided space. Answer written elsewhere will not be evaluated. Overwritting will be considered as unattempted.

1. An equivalent double integral, with the order of integration reversed, of $\int_0^2 \int_0^{4-y^2} y\,dx\,dy$ is $\boldsymbol{\int_0^4 \int_0^{\sqrt{4-x}} y\,dy\,dx}$.

2. For the space curve, $\mathbf{r}(t) = (\cosh t)\mathbf{i} - (\sinh t)\mathbf{j} + t\mathbf{k}$, the unit tangent vector at $t = 0$ is $\boldsymbol{\frac{-\mathbf{j}+\mathbf{k}}{\sqrt{2}}}$.

3. The interval of convergence for the series $\sum_{n=1}^{\infty} \sqrt[n]{n}(2x+5)^n$ is $\boldsymbol{-3 < x < -2}$.

4. The set of points (x, y), where the function $f(x, y) = \sqrt{x^2 + y^2 - 4}$ is continuous, is **closed but not open.** *(open but not closed/closed but not open/both open and closed/neither open nor closed)*

5. The critical point of the function $f(x, y) = x^2 + y^2 + 3x + 2y + 5$ is a point of **local minima.**
(local maxima/local minima/saddle point/no conclusion)

6. The linearization $L(x, y)$ of the function $f(x, y) = (x + y + 2)^2$ at the point $(2, 1)$ is $\boldsymbol{10x + 10y - 5}$.

7. Let D be the region in the first octant that is bounded below by the cone $\phi = \pi/4$ and above by the sphere $\rho = 2$. The iterated integral for the volume of D in terms of cylindrical coordinates is $\boldsymbol{\int_0^{\pi/2} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta}$.

8. The flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the sphere $x^2 + y^2 + z^2 = a^2$ in the outward direction is $\boldsymbol{4\pi a^3}$.

9. The flow of the velocity field $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j}$ along upper half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ in the xy-plane is $\mathbf{0}$.

10. The area of the triangular surface, cut from the plane $2x + 6y + 3z = 6$ by the coordinate planes, in the first octant is $\boldsymbol{\frac{7}{2}}$.

END

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PART-II (OPEN BOOK)

Time: **135** Minutes | Max. Marks: **90**

Important Instructions

- This question paper consists of two parts: **PART-A** (Three questions) and **PART-B** (Four questions).
- Answer the questions of **PART-A** and **PART-B** in two separate answer sheets. Also, write Part-A and Part-B on top right corner of respective answer sheet.
- Answer each question from the new page. Each subpart of a particular question should be answered in continuation.
- Calculator is not allowed.

PART-A

A1. (a) Compute the value of

$$\lim_{n\to\infty} \frac{m(m-1)\dots(m-n+1)}{(n-1)!}x^n,$$

when $|x| < 1$ and m is any real number. **[7]**

(b) Using Integral test, check the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}.$$ **[6]**

A2. Re-parametrize the curve $\mathbf{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\mathbf{i} + \left(\frac{2t}{t^2+1}\right)\mathbf{j}$ in terms of s (arc length) from the point $(1, 0)$ in the direction of increasing t. Hence, identify the curve. **[13]**

A3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f(x,y) = \begin{cases} \frac{xy^3}{x+y^2}, & (x,y) \neq (0,0); \\ 0, & (x,y) = (0,0). \end{cases}$$

(i) Find $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ at $(0, 0)$. **[8]**

(ii) Examine the continuity of $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ at $(0, 0)$. Justify. **[4]**

PART-B

B1. (a) Using the method of Lagrange multipliers, find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$. **[7]**

(b) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$. **[6]**

B2. Let R be the region bounded by lines $y = x$ and $y = x + 1$ and by the hyperbolas $y = 1/x$ and $y = 2/x$. Use the transformations $u = y - x$ and $v = xy$ to evaluate the double integral

$$\iint_R (x + y)\, dx\, dy. \qquad \textbf{[13]}$$

B3. (a) Use Green's theorem to calculate $I = \oint_C -x^2 y dx + xy^2 dy$, where C is circle of radius 2 centered at origin, oriented counterclockwise. Hence, give the conclusion about the existence of the potential function f such that $\nabla f = -x^2 y\mathbf{i} + xy^2\mathbf{j}$. Justify your answer. **[9]**

(b) Find the circulation of the vector field $\mathbf{F} = z\mathbf{i} + y^2\mathbf{j} + x\mathbf{k}$ around the curve

$$\mathbf{r}(t) = (\cos t)\mathbf{i} + (4\sin t)\mathbf{j}; \quad 0 \le t \le 2\pi. \qquad \textbf{[4]}$$

B4. Set up and evaluate the surface integral for the work done by the force $\mathbf{F}$ in moving a particle along C in the counterclockwise direction when viewed from above, where $\mathbf{F} = xy^2\,\mathbf{i} + \mathbf{j} + z\,\mathbf{k}$ and C is the curve of intersection of the upper hemisphere $x^2 + y^2 + z^2 = 4, z \ge 0$ with the cylinder $(x - 1)^2 + y^2 = 1$. **[13]**

****END****